A Primer on the Measurement of Net Stocks, Depreciation, Capital Services, and Their Integration.

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1.0 Forward

This manual is intended to be a “primer” on the rudiments of the methodologies used to estimate capital stocks and the value of their services by statistical agencies. It explains the key parts of methodologies used to estimate wealth stock estimates through the use of numeric examples and avoids, as much as possible, the use of mathematical equations. Numerical examples are emphasized because many people find it easier to understand the material in this form. In fact, this author initially learned the neoclassical capital accounting framework with the aid of numerical examples provided to him by Professor Frank Wykoff.

The primer gives examples using both straight-line and geometric age-price profiles. Examples for straight-line depreciation are given because it used to be the primary method of the Bureau of Economic Analysis (BEA), is widely used in business accounting and in European official statistics, and is easy for novices to understand. Examples for geometric-depreciation are given because that is now BEA’s primary method and it is widely employed elsewhere.

The primer also gives an elementary exposition of how capital services can be estimated. This is done because the two are interrelated. The Bureau of Labor Statistics (BLS) publishes official estimates of capital services. In constructing these estimates BLS also estimates wealth stocks that are consistent with these flows; these stocks differ from the official wealth stocks published by BEA. There are now attempts to integrate the flow estimates from BLS with the stock estimates from BEA.[1] But, there is little material published on the conceptual problems faced in attempting this integration.

Obtaining a basic understanding of these conceptual problems is extremely important. If you do not understand how estimates of the capital stock and capital services are related to each other, you cannot fully understand either of the two. This primer attempts to give readers this understanding without requiring that they learn all of the details of the underlying methodologies.

The primer examines two possible approaches to stock measurement. One, used by BEA, employs the use of fixed age-price profiles. The second, used by BLS, employs fixed age-efficiency profiles. Prior to 1997, BEA used a capital stock methodology that employed straight-line depreciation, distributions of finite service lives about a mean taken from the work of Robley Winfrey, and fixed-weighted price indexes. The conceptual foundations of this methodology were discussed in Young and Musgrave (1980). Its details were presented in BEA (1993). BEA now uses a capital stock methodology that uses geometric depreciation for most goods as well

[1] See Harper et. al. (2009).

as chain-weighted price indexes. The methodology was introduced in Katz and Hermann (1997) and used depreciation schedules recommended in Fraumeni (1997). The methodology was described in detail in BEA (2003), which presented its core part in equation form.

The alternative capital stock methodology, which is based on the use of age-efficiency profiles generated by a hyperbolic or beta-decay function, was developed by Dr. Noel Roy of Jack Faucett Associates (1967) and further developed in BLS (1970). This methodology was well explained in BLS (1979), where it was used in capital stock estimates for input-output industries. The report discussed the role of maintenance and repair expenditures on depreciation in deciding which parameter values were most appropriate for the model. This topic was developed in detail in Faucett (1980). A similar methodology was developed for use in multifactor productivity estimates and was presented in BLS (1983).

The relationship between an asset's age-efficiency profile and its age-price profile was presented in Faucett (1980). Katz (1982) presented the age-price profiles that are derived from different age-efficiency profiles; similar material is ubiquitous.

2.0 Introduction

This manual explains the basic methodology used by the Bureau of Economic Analysis (BEA) to estimate the value of net stocks and depreciation. It also explains some of the central points of the methodology used by the Bureau of Labor Statistics (BLS) and others to estimates the services of the capital goods that comprise BEA's stocks. The manual shows how, in concept, the methodologies are related to each other. It also sheds light on the problems involved in trying to develop an integrated system that combines these estimates of stocks and flows into a coherent whole.

The manual is organized as follows. Chapter 1 provides some introductory material. Chapter 2 explains the basic concepts and definitions. Chapter 3 explains the basic BEA capital stock methodology using an example assuming straight-line depreciation and an explicit retirement distribution and an example assuming geometric depreciation without an explicit retirement distribution. Chapter 4 explains how capital services can be estimated from either wealth or productive stocks. Calculations are shown both for the example assuming straight-line depreciation and for the example assuming geometric depreciation. Chapter 5 discusses if it is possible to estimate capital stocks and services in an integrated system that uses a common set of assumptions. Chapter 6 presents a summary and some concluding remarks. Appendix A presents a derivation of the user cost of capital measure of the implicit rental value of an asset's services. Appendix B shows how implied age-price profiles can be estimated from the corresponding age-efficiency profiles. Appendix C proves that if depreciation is not geometric, a change in the real own interest rate will cause either an asset's age-price profile or its age-efficiency profile to change.

2.1 Uses of the Estimates

The depreciation estimates are generally synonymous with consumption of fixed capital (CFC), a charge for the using up of capital that is one of the costs incurred in the production of GDP. In the National Income and Product Accounts (NIPAs) that are estimated by BEA, the CFC estimates are used to derive estimates of net domestic product and net domestic income, as well as net investment and economic profits. The net stock estimates are measures of wealth that are recorded in the balance sheets of the Federal Reserve Board Flow of Funds Accounts (FFAs). These data are also recorded in the Integrated Macroeconomic Accounts for the United States, which are part of an interagency effort to further harmonize data from the NIPAs and the FFAs. In addition, BLS produces estimates of the services of capital goods, which are used in studies of productivity. Depreciation is a major component of the cost of capital services, which raises the questions of consistency between the way depreciation is calculated for purposes of productivity measurement and the way it is measured in the NIPAs.

2.2 Valuation

BEA publishes estimates based on three different valuations: historical cost, current cost, and real cost (chained-dollars). The historical-cost estimates use the prices that were in effect when each asset was first purchased. These estimates are similar to those that appear in company reports. They are used in such accounting measures because they are "conservative." Such measures are used by the IRS in determining one's income tax liability. They are also used in the measures of profits reported by the IRS. Historical-cost estimates are generally not used in economic analysis because they are obtained by summing up values measured in prices of different time periods.

Current-cost measures of the value of assets use prices of the current year. The current-cost measure of an asset's value is important because, in equilibrium, it represents the discounted present value of the services that the asset will yield in the future.

Real-cost measures of the value of assets use prices of the base or reference year. Because these measures are adjusted for the effects of inflation, economists find them particularly useful in economic analysis.

3.0 Basic Concepts and Definitions

Fixed assets are produced assets that are used repeatedly, or continuously, in processes of production for more than one year. (Produced assets are nonfinancial assets that have come into existence as outputs from a production process.)

BEA's **net capital stock** is defined as the value of the stock of fixed assets after adjustment for depreciation. It is a measure of wealth. In principle, the current-price value of this wealth should equal the amount that it could be sold for on resale markets.

Depreciation is defined as the decline in the value of stocks of fixed assets due to wear and tear, obsolescence, accidental damage, and aging. It is often confused with the decline in the value of capital goods from the beginning to the end of a year. Actually, current-price depreciation is equal to this decline in market value less any capital gains on the value of the asset due to inflation in its price, i.e., holding gains, and any other changes in the (physical) volume of the stock of the assets.

Estimates of the value of capital stocks may be made using either the direct volume method or the perpetual inventory method. Under the **direct volume method**, there is a direct count of the number of physical units of each type of capital. The value of a unit of each type of capital is determined in a separate set of calculations. For example, we could count the number of houses in a census and then value them using prices obtained from a census, tax assessments, or data on sales of used homes. Under the **perpetual inventory method**, the net stock and depreciation are indirectly estimated by cumulating past investment flows. That is, the net stock and depreciation of any given type of asset are both weighted (but different) summations of past gross investment in that asset.

For the most part, BEA uses the perpetual inventory method.[2,3] Consequently, the remainder of this primer assumes that this method is being used. Let us assume that a new unit of a given asset, a widget, has a purchase price of 100 at the beginning of year t-1 and that it has a service life of 4 years. Let us further assume that identical widgets were produced in prior years and will continue to be

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[2] BEA estimates the net stock and depreciation of autos using the direct volume method. It also uses the direct volume method to estimate the value of housing services, i.e., space rent, although it uses the perpetual inventory method to estimates the net stock and depreciation of housing.

[3] BEA's methodology for estimating capital stocks and depreciation is found in U.S. Department of Commerce, Bureau of Economic Analysis (2003). A detailed mathematical example of the methodology is given in an appendix to Katz (2008).

produced in future years. We assume that in year t-1 the asset's "apparent" age-price profile declines linearly in a straight-line manner to zero. In other words, at the beginning of year t-1, the price of a 1-year-old widget is 75, the price of a 2-year widget is 50, and the price of a 3-year-old widget is 25 as shown in table 1 below.

Table 1 - Price of Asset at the Beginning of the Year, by Age of Asset

Year	t-1
Age	
0-years old (new)	100
1-year old	75
2-years old	50
3-years old	25

Note that different aged assets come from different vintages of the asset. The 1-year old asset was produced and entered the stock in year t-2, the 2-year old asset was produced and entered the stock in year t-3, etc. The series of prices shown in table 1 determine the asset's **"apparent" age-price profile.** This profile shows how the asset's price varies between different ages at a given point in time, i.e., it gives the relative prices of assets that differ solely with respect to their age. These relative prices of older assets are often expressed as fractions of the price of a new asset.

Assume, for example, that the price of a new asset has inflated to a value of 104 at the beginning of year t and that the apparent age-price profile in this year also declines linearly to zero. Then, the prices of assets of various ages at the beginning of year t-1 and year t will be as shown in table 2.

Table 2 – Current-price Value of Asset, in (Current) Prices of the Beginning of the Year, by Age and Year

Year	t-1	t
Age		
0-years old (new)	↓ 100→	104↓
1-year old	75→	78
2-years old	50	52
3-years old	25	26

Now, the asset that was new at the beginning of year t-1 is one year old at the beginning of year t. Its market value has declined from 100 to 78 during year t-1. This decline in market value can be decomposed into depreciation less capital gains in several different ways. [4] Here, depreciation is defined as the difference in price at a given instant in time between two assets that differ solely in that one is a year older than the other. This definition is employed by economists who are primarily interested in measuring the services of capital goods and productivity. It is used here because it is generally consistent with the concepts used in the national accounts.[5]

To see how this decomposition works, let us first measure depreciation using beginning-of-year prices as in the work of Hulten and Wykoff (1980). Then, as shown by the red arrows in table 2, the decline in the price from 100 to 75 (i.e., 25) is depreciation while the increase in the price from 75 to 78 is the capital gain. Alternatively, suppose we measure depreciation using end-of-year prices as Dale Jorgenson (1989) does. Then, as shown by the green arrows, the increase in the value of the asset from 100 to 104 is the capital gain on the asset while depreciation is the decline in its price from 104 to 78 (i.e., 26). BEA measures depreciation using average prices during the year. This yields estimates that are essentially an average of those obtained from the Jorgenson and Hulten-Wykoff methods. (In other words, at average prices of the year, a new asset would have a value of 102 while a 1-year old asset would have a value of 76.5. Depreciation of the asset, measured in current prices, would then be equal to 102 - 76.5 or 25.5.)

4.0 Basic BEA Capital Stock Methodology

Over the decades, BEA has taken two basic approaches to measuring stocks using the perpetual inventory method. Prior to 1997, estimates were made using explicit asset retirement distributions and straight-line depreciation. Since 1997, the stocks for most assets have been estimated using geometric depreciation without

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[4] It is not necessary to separate the total decline in market value into these two components in order to estimate capital services. However, the depreciation component is recorded in measures of current income while the capital gain component is not.

[5] The national accounts definition of depreciation uses data for only a single vintage of an asset. It is assumed that an asset's age-price profile remains fixed over its service life in both current and constant prices. In a few instances, BEA assumes that different vintages of the same type of asset have different age-price profiles. These exceptions include computers, computer peripheral equipment, and some assets where the mean life is assumed to be different in different vintages. In these instances, BEA treats the different vintages of the asset as if they were actually different types of assets. Nowhere in its depreciation estimates does BEA subtract the price found on one profile from a price found on a different profile. However, in some empirical studies, depreciation is estimated by subtracting prices from profiles for two different vintages of the asset. Because the age-price profiles in these studies are permitted to vary from vintage to vintage, the two profiles used in the construction of these estimates may not be identical.

any explicit retirement distribution. This section explains the two approaches using stylized examples that capture their essence.

4.1 Basic Methodology With Linear Depreciation and an Explicit Retirement Distribution

BEA's basic perpetual inventory method for estimating stocks and depreciation makes assumptions regarding assets' actual age-price profiles. An asset's **"actual" age-price profile** measures its price at various ages relative to its price when new, both prices being measured in terms of constant prices of the reference year.[6] Operationally, the profile for a given type of asset describes the pattern of how, in the absence of inflation, the price of an asset of that type declines as it ages.

To get a better understanding of what this means, let us expand table 2 and express all prices in terms of the prices in effect at the beginning of year t, which we shall take as the reference year. When we do this, the table will look as shown below in table 3.

Table 3 - Value of Asset in Constant Prices of the Beginning of Year t, by Age and Year

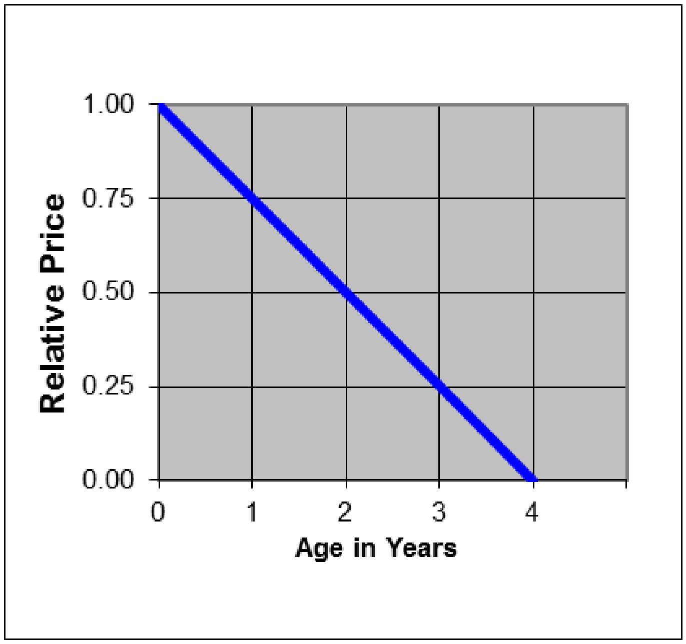
Year	t-1	t	t+1	t+2
Age				
0-years old (new)	**104**	104	104	104
1-year old	78	**78**	78	78
2-years old	52	52	**52**	52
3-years old	26	26	26	**26**

Constant-price measures of the asset's actual value are shown in red along the main diagonal. It is these values that constitute the asset's actual age-price profile. All other values in the table correspond to actual values of *other* vintages of the asset. As noted above, each point on the asset's age-price profile shows its price at a given age relative to its price when it was new. Given the values

[6] In this primer, the term **constant-price** is used to denote "real" estimates that are adjusted for inflation using a fixed-weighted price index. The term **chained-price** is used to denote real estimates that are adjusted for inflation using a chain-weighted price index. For estimates of capital stocks and flows involving only a single type of asset, the two valuations are identical. For estimates involving aggregates composed of two or more types of assets, the two valuations generally differ. When chained-price estimates are expressed in level form they are described as being made in chained dollars. This paper uses the adjectives "chained-price" and "chained-dollar" synonymously.

shown in table 3, the asset's age-price profile is a straight-line that declines linearly to zero as shown in figure 1.[7] BEA actually uses

Figure 1 - Age-price Profile for Data in Table 3



straight-line patterns of depreciation for only a few assets. Most assets are assumed to have age-price profiles that decline at constant geometric rates. The use of geometric profiles makes it much easier to compute estimates. Also, as we shall see later in the primer, estimates made using geometric profiles have certain properties that make it easier to achieve integration between capital stocks and capital services. These properties are not shared with any profiles that are not geometric.

In BEA's application of the perpetual inventory method, it is assumed that an asset's (actual) age-price profile remains fixed over its service life. It is this fixed profile that is used to estimate values of depreciation and the net stock in both current and constant prices. Methodologies that employ the "difference-in-price" measure of depreciation construct values of depreciation and the net stock using apparent rather than actual age-price profiles. In many circumstances, the valuation of net stocks and depreciation will be the same regardless of whether the underlying methodology is based on the use of fixed age-price profiles or apparent age-price profiles. In fact, the first few examples in this primer are constructed so that the

[7] In BEA's actual methodology, the constant-price net stock is measured using average prices during the year, which are essentially mid-year values. Beginning-of-year prices are used in the examples given here because it simplifies the exposition.

two methodologies will yield the same results. But, as we shall later see, when interest rates change, the two methodologies may produce results that differ from each other.

Given BEA's assumption of a fixed age-price profile, the value of that part of the stock of the asset in a given year, k, resulting from investment in year z is estimated as follows. Nominal investment in the asset in year z is converted to constant-price investment by dividing the nominal value by the appropriate constant-quality price index for the asset (i.e., the asset's price in year z divided by its price in the reference year). The constant-price values are treated as if they were quantity (or volume) measures.

We obtain the depreciated value of this investment in constant prices by multiplying its un-depreciated value by the point on the age-price profile for the relevant age of the investment (k-z). This will give us the depreciated values that are shown on table 3. These values are converted back into current prices by reflating, i.e., multiplying the constant-price value by the value for the given year of the price index for the investment.

BEA constructs estimates of depreciation using a similar methodology. The constant-price value of depreciation in year k resulting from investment in year z is obtained by multiplying constant-price investment in the asset in year z by the difference between the value on the age-price profile that the investment has at the beginning of the year and the value on this profile that it has at the end of the year. The current-price value of this depreciation is obtained by reflating the constant-price measure using the average value of the appropriate reflator in year k.

Let us trace this calculation through in terms of the example shown in tables 2 and 3. Suppose that the asset results from investment in year t-1 (i.e., z=t-1) and that we are valuing it in year t (i.e., k = t). Investment in year t-1 consisted of one unit of the asset. Because year t is the reference year, this unit had a value of 104 in constant prices when it was new. In year t, the asset is one year old (i.e., k-z=1) By multiplying 104 by the appropriate value from the age-price profile we find that in constant prices the asset has a value of 78 at the beginning of year t as shown in table 3. Using data from table 2, we see that the price index for new units of the asset has a value of 104/104 at the beginning of year t. Multiplying 78 by this, we obtain a current price value of 78 for the asset as of the beginning of year t.

Constant-price depreciation on the asset in year t-1 is estimated by multiplying the constant-price value of the asset when it first entered the stock, 104, by the change in its age-price profile from the beginning of year t-1 to the beginning of year t, i.e., by 1.0 -.75 =

.25, to obtain a value of 26. (This equals the difference between the asset's constant-price value at the beginning of year t-1 and its value at the beginning of year t.) To reflate this value to current prices, we obtain the average value of the price index for year t-1. This is found by taking the average of the values for the beginning and end of the year, i.e., the average of 100/104 and 104/104. This average value is, therefore, 102/104. Multiplying this average value by 26, we obtain a value of 25.5 for current-price depreciation.

4.1.1 Service Lives

Perhaps the most important step in the implementation of the perpetual inventory method is the determination of the service lives that are assumed for the various types of capital goods. The selected service lives have a major impact on the values of the stock and depreciation. It is important to remember several facts regarding these lives. First, these are essentially forecasted values of future service lives; past values of these lives are only relevant to the extent that they help us to better forecast future lives.[8] Second, we are dealing with **economic lives** and not physical lives. Capital goods are retired when it is no longer profitable to use them. This may be long before they are broken or need extensive repairs.

Perhaps the most important determinant of these lives is expected obsolescence. Expected or normal obsolescence is part of measured depreciation. There is no way to separate out how much of total depreciation is due to this obsolescence. It is impossible to determine why a good is retired from the stock and it is irrelevant for our accounting. But, it is essential to know when it will be retired. Obsolescence is not just due to the introduction of technologically superior goods. An increase in the relative cost of repairing and maintaining used capital goods may result in a decrease in the number of years that it is profitable to use them. When service lives decrease because of such changes in economic conditions rather than because of increases in physical wear and tear, one may regard this as an effect of increased obsolescence. Similarly, large increases in the price of gasoline can cause gas-guzzling cars to suffer some obsolescence.

4.1.2 Asset Retirement Distributions

An **asset retirement distribution** shows how the service lives of a **cohort of investment** (all assets purchased in a given year) are distributed around the cohort's assumed mean value. All assets are

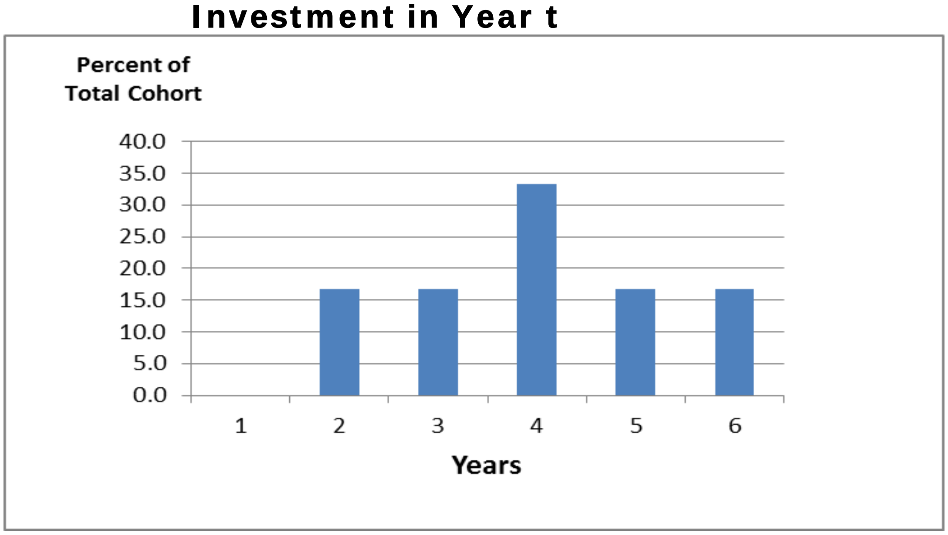
[8] Earlier we assumed that all assets reach the service life that the estimator initially expects that they will have and then the assets expire. At some later point the estimator may revise the initial assumption of what the service life is and then recalculate the entire time series for the stock of this asset.

assumed to reach the end of their assumed lives and have a value of zero when they are discarded (retired) from the stock. Suppose that the mean life for computer printers is 10 years and that we know what the discard distribution is. This means that we know that x percent of the cohort will have a life of 12 years, y percent will have a life of 11 years, etc. We are making estimates for the aggregate depreciation of an entire cohort of printers and it is immaterial whether we can or cannot identify which specific printers will have the 12-year life.

There are two general ways to estimate an aggregate depreciation pattern for an entire cohort of investment. One is to directly assume a specific aggregate pattern. The second is to assume a specific retirement distribution and a specific depreciation pattern for individual assets. The first method is generally used with geometric depreciation. That is, the strictly geometric pattern is generally assumed for an entire cohort of assets. Individual assets within the cohort need not exhibit a strictly geometric pattern of declines in value and may be discarded at the end of a finite life. Conversely, in making capital stock estimates, the straight-line pattern of depreciation has generally been assumed to apply to individuals assets and an asset retirement distribution has been used in conjunction this pattern to obtain estimates of aggregate depreciation. Consequently, users of depreciation estimates need to understand what asset retirement distributions are and how they are being used in order to be able to compare estimates based on the two approaches.

To see how the two approaches are related to each other, consider the following example. Let us assume that six widgets are purchased in year t. Let us further assume that the service lives of the six widgets are, respectively: 2, 3, 4, 4, 5, and 6 years. Then, the retirement distribution for the widgets is as shown in figure 2.

Figure 2 - Distribution of Assumed Service Lives for Investment in Year t



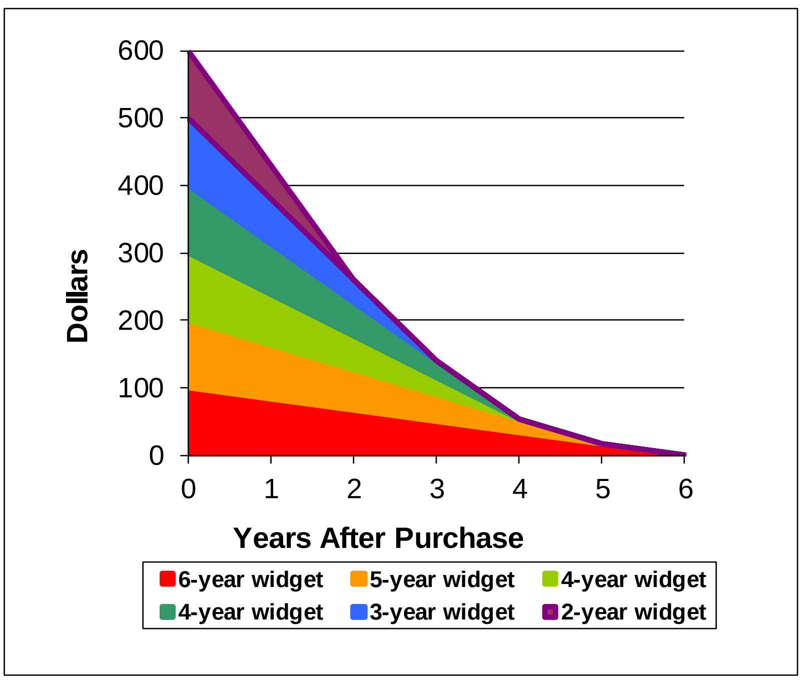
Let us further assume that each of the assets had a constant-price value of 100 when it was purchased at the beginning of year t and that this constant-price value declines linearly to zero over the asset's service life. In other words, the age-price profile for each asset is assumed to follow a straight-line pattern. Then, the constant-price values for each asset at the beginning of years t through t+6 are given by those shown in table 4.

Table 4 - Distribution of Assumed Future (Beginning-of-year) Values of Assets Purchased in Year t, in Constant Prices

Year	t	t+1	t+2	t+3	t+4	t+5	t+6
2-year widget	100.0	50.0	0.0	0.0	0.0	0.0	0.0
3-year widget	100.0	66.7	33.3	0.0	0.0	0.0	0.0
4-year widget	100.0	75.0	50.0	25.0	0.0	0.0	0.0
4-year widget	100.0	75.0	50.0	25.0	0.0	0.0	0.0
5-year widget	100.0	80.0	60.0	40.0	20.0	0.0	0.0
6-year widget	100.0	83.3	66.7	50.0	33.3	16.7	0.0
Total	600.0	430.0	260.0	140.0	53.3	16.7	0.0

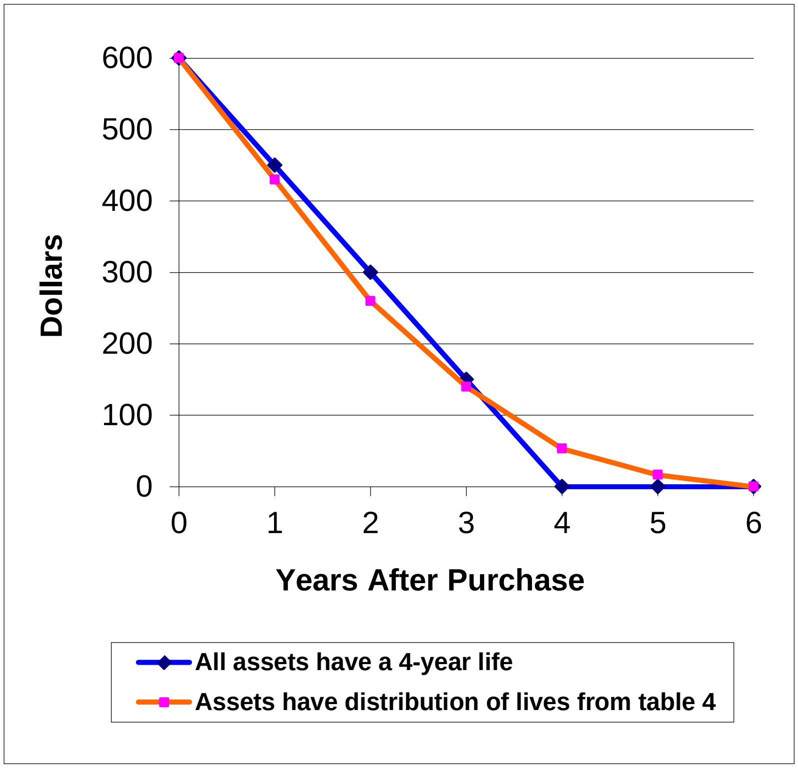
In table 4 we sum the values of the six widgets in each year to obtain a value for net stock resulting from investment in year t. Figure 3 shows how much each widget's value contributes to the value of the total stock. The stock's value declines in a manner that is nearly

Figure 3 - Net Stock of Investment Cohort with Assumed Retirement Distribution



curvilinear. Figure 4 shows the value of this stock and that of the stock that would have resulted if each of the widgets had a life of 4 years. Obviously, the value of this latter stock declines linearly in a straight-line manner. The first stock's value declines at a faster pace (it has a higher depreciation rate) during the first two years after the

Figure 4 - Net Stock of Investment Cohort for Two Retirement Distributions



purchase was made and it declines at a slower pace thereafter. These results illustrate that the shape of the retirement distribution has a major impact on the aggregate depreciation pattern.

4.2 Basic Methodology With Geometric Depreciation and No Explicit Retirement Distribution

BEA assumes that most categories of fixed investment have age-price profiles that follow a strictly geometric pattern. The geometric pattern is applied to an entire cohort of investment rather than just the investment for a single asset. While the tables shown below are presented as if they were for a single asset, they should be interpreted as if they were for the entire cohort of assets. One way to think of this is that these are the estimates that would hold if all assets in the cohort had identical service lives and profiles. Because the use of the geometric pattern results in de facto infinite service lives for many assets in the cohort, numerical examples using this pattern may be somewhat more difficult to follow than those based on the straight-line method. In practice, the use of geometric depreciation actually results in a significant reduction in the complexity of the calculations.

As in section 3.0, let us assume that a new unit of a given asset has a purchase price of 100 at the beginning of year t-1 and that the widget has a service life of 4 years. Let us further assume that identical widgets were produced in prior years and will continue to be produced in future years. We assume that in year t-1 the asset's "apparent" age-price profile declines in a strictly geometric manner at a 1.6-declining-balance rate. This means that in the first year of the asset's life, depreciation on it will be 1.6 times the amount if would have been had the asset been depreciated using the straight-line method. With the straight-line method, depreciation in the first year would be 1 divided by the service life or ¼. In other words, depreciation in the first year occurs at an annual rate of 25 percent. Consequently, we are assuming that the asset depreciates at an annual rate of 1.6 times this, i.e., at a rate of 40 percent per annum. Consequently, at the beginning of year t-1, the price of a 1-year-old widget is 60, the price of a 2-year widget is 36, the price of a 3-year-old widget is 21.6, etc., as shown in table 5 below. When the asset is 10 years old it has a value of only 0.6. Technically, the asset is never fully depreciated. It remains in the stock and continues to depreciate as its value declines to ever smaller values over time.

In table 5, assets of different ages come from different vintages of the asset. The series of prices shown in this table determine the asset's “apparent" age-price profile.

Table 5 - Price of Asset at the Beginning of the Year, by Age of Asset

Year	t-1
Age	
0-years old (new)	100
1-year old	60
2-years old	36
3-years old	21.6
…………………….	…….
10-years old	0.6

Assume, for example, that the price of a new asset has inflated to a value of 104 at the beginning of year t and that the apparent age-price profile in this year also declines at a constant geometric rate of

40 percent per annum. Then, the prices of assets of various ages at the beginning of year t-1 and year t will be as shown in table 6.

Table 6 – Current-price Value of Asset, in (Current) Prices of the Beginning of the Year, by Age and Year

Year	t-1	t
Age		
0-years old (new)	100	104
1-year old	60	62.4
2-years old	36	37.44
3-years old	21.6	22.46
…………………….	…….	……………
10-years old	0.60	0.63

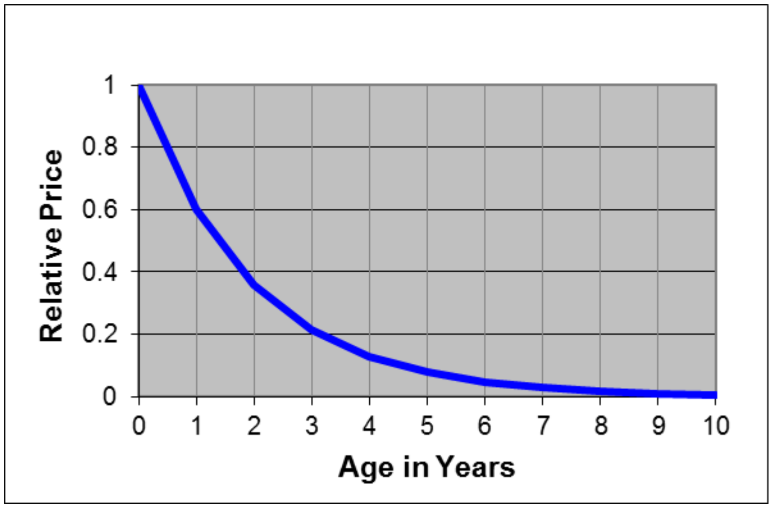
To get a better understanding of the difference between the asset's actual age-price profile and its apparent age-price profile, let us expand table 6 and express all prices in terms of the prices in effect at the beginning of year t, which we shall take as the reference year. When we do this, the table will look as shown below in table 7.

Table 7 - Value of Asset in Constant Prices of the Beginning of Year t, by Age and Year

Year	t-1	t	t+1	t+2		t+9
Age						
0-years old (new)	**104**	104	104	104	……….	104
1-year old	62.4	**62.4**	62.4	62.4	……….	62.4
2-years old	37.44	37.44	**37.44**	37.44	……….	37.44
3-years old	22.46	22.46	22.46	**22.46**	……….	22.46
………………………	……….	……….	……….	……….	……….	……….
10-years old	0.63	0.63	0.63	0.63	……….	**0.63**

Constant-price measures of the asset's actual value are shown in red along the main diagonal. All other values in the table correspond to actual values of other vintages of the asset. As noted above, each point on the asset's age-price profile shows its price at a given age relative to its price when it was new. The asset's age-price profile is constructed by dividing each number on the main diagonal by 104, the price of the asset when new. Given the values shown in table 7, the asset's age-price profile has values that decline at a constant geometric rate as shown in figure 5. (The values in the profile are truncated after 10 years; technically, the asset has an infinite life.)

Figure 5 - Age-price Profile for Data in Table 7



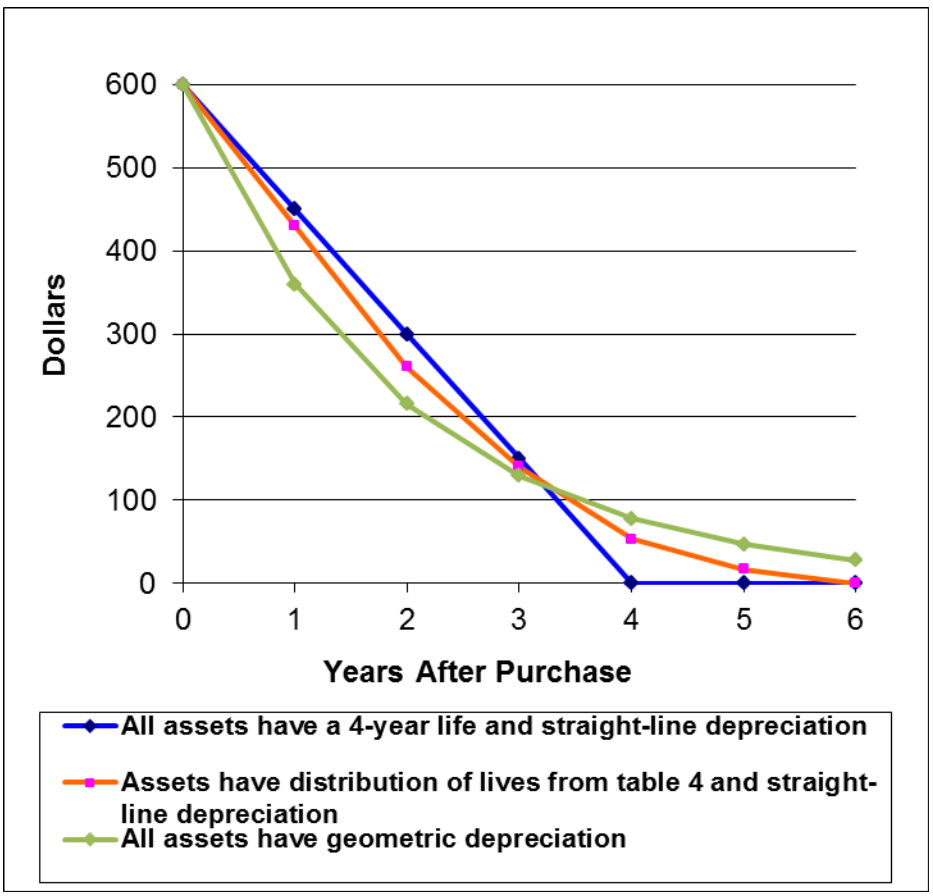
We can illustrate the steps involved in BEA's methodology for estimating depreciation using the data for the example shown in tables 6 and 7. Suppose that the asset results from investment in year t-1 and that we are valuing it in year t. Investment in year t-1 consisted of one unit of the asset. Because year t is the reference year, this unit had a value of 104 in constant prices when it was new. In year t, the asset is one year old. By multiplying 104 by the appropriate value from the age-price profile we find that in constant prices the asset has a value of 62.4 at the beginning of year t as shown in table 7. Using data from table 6, we see that the price index for new units of the asset has a value of 104/104 at the beginning of year t. Multiplying 62.4 by this, we obtain a current price value of 62.4 for the asset as of the beginning of year t.

Constant-price depreciation on the asset in year t-1 is estimated by multiplying the constant-price value of the asset when it first entered the stock, 104, by the change in its age-price profile from the beginning of year t-1 to the beginning of year t, i.e., by 1 -.60 = .40, to obtain a value of 41.6. (This equals the difference between the asset's constant-price value at the beginning of year t-1 and its value at the beginning of year t.) To reflate this value to current prices, we obtain the average value of the price index for year t-1. This is found by taking the average of the values for the beginning and end of the year, i.e., the average of 100/104 and 104/104. This average value is, therefore, 102/104. Multiplying this average value by 41.6, we obtain a value of 40.8 for current-price depreciation.

These results can be compared with those we obtained earlier. In this section we estimated aggregate depreciation for an entire cohort of investment without the use of a retirement distribution. In section 4.1 we estimated aggregated depreciation using straight-line depreciation for individual assets and a specific retirement distribution.

That is, we assumed that six widgets were purchased in year t and that their service lives were, respectively: 2, 3, 4, 4, 5, and 6 years. The mean of this distribution is 4 years. In figure 6 we compare three aggregate depreciation patterns. In the pattern shown in orange, the assets have the distribution of service lives assumed here and age-price profiles that decline linearly to zero. In the pattern shown in blue, all assets have the mean life of 4 years and have age-price profiles that decline linearly to zero, i.e., they are depreciated using the straight-line method. Because the pattern is identical for all assets, the aggregate depreciation pattern is also a straight-line. In the pattern shown in green, aggregate depreciation occurs at a constant geometric rate of 40 percent per annum, i.e., the rate that is used throughout this section.

Figure 6 - Net Stock of Investment Cohort for Three Retirement Distributions



The example highlights certain comparative properties of geometric depreciation. In the first few years after an investment is made, the stock will have a lower value than with estimates based on a finite service lives. Conversely, the value of stock for years well after the investment was made will always be greater using geometric depreciation because the aggregate age-price profile has an infinite tail while estimates using the other methods will have their values go to zero after a finite number of years.

4.3 A Key Property of BEA's Capital Stock Methodology

In BEA's perpetual inventory methodology, with an exception that is described below, all assets are assumed to reach the end of their assumed lives, at which point they are fully depreciated, have a zero value, and are discarded. Assets are not prematurely discarded; in other words, **all discards have a value of zero.** This treatment helps to ensure that **over the lifetime of each asset, constant-price depreciation will sum up to the asset's initial purchase price in constant prices**. This property is a cornerstone of BEA's capital accounting framework. It is this property that anchors the total value of estimates of net stock and net investment and makes them meaningful. With it there is a stock-flow identity in constant-prices; that is, the value of the stock at the end of the year is equal to the stock at the beginning of the year plus gross investment less depreciation. Consequently, a value of constant-price net investment that is greater than zero indicates that the constant-price net stock in increasing.

An exception to this is that catastrophic damage and war losses are not treated as depreciation but as "other changes in the volume of assets," which are essentially adjustments to balance sheets that do not affect the income and product accounts. These adjustments are made so that large changes in the net stock that do not result from economic processes do not have a direct effect on measured economic income and product.

4.4 BEA's Depreciation Rates

BEA's depreciation rates are largely taken from recommendations made by Barbara Fraumeni (1997). She used the results of an empirical study by Hulten and Wykoff (1981) to recommend specific rates of geometric depreciation for most types of assets. For computers, computer peripheral equipment, and autos, she recommended that BEA use specific schedules found in empirical studies. For nonresidential equipment for which studies were not available, she recommended that BEA use geometric depreciation at a declining-balance rate of 1.65 as its default option. In other words, if an asset has a service life of 10 years, with the straight-line method it has a depreciation rate of 1/10 or 10 percent in its first year. With a declining balance rate of 1.65, we would multiply the 10 percent by 1.65 and obtain a constant geometric rate of 16.5 percent. She recommended a default declining-balance rate of 0.91 for structures. These default rates are used when no specific information is available from empirical studies regarding the proper declining-balance rate.[9,10] Prior to 1996, BEA estimated depreciation using age-price profiles that declined linearly to zero in a straight-line manner in conjunction with asset retirement distributions of service lives that were essentially discrete approximations to normal curves with finite tails.

4.5 Aggregation Over Different Types of Assets

Earlier in this chapter we showed how stock and depreciation estimates could be made in current and constant prices, i.e., the estimates are expressed in current and constant dollars. This methodology holds for estimates made for any single type of asset. But, how do we make estimates for aggregates that comprise more than one type of asset? For estimates expressed at current prices, there is no problem. These estimates are strictly additive and all we need to do is to sum up the estimates for each of the individual types of assets that comprise the aggregate. For real estimates expressed in constant prices of a given reference year, there is also no problem. They are also strictly additive. But, there is a problem when real estimates are expressed in chained prices as these are not strictly additive. Chained-price estimates are featured by BEA, however, because they remedy certain deficiencies of the constant-price estimates.

Chained-price measures are constructed by chaining together estimates of year-to-year growth rates. Thus, the growth rate for years t to t+1 is multiplied by the growth rate of years t+1 to t+2, etc. in this process. The growth rates for individual years are constructed using the Fisher price index, which is the geometric mean of a Laspeyres price index and a Paasche price index. For growth rates from year t to year t+1, the Laspeyres index uses weights for year t while the Paasche index uses weights for year t+1. These indexes are more fully described, together with worked out examples, in chapter 4 of BEA's NIPA Handbook (2011).

The use of chain-type indexes has a number of important advantages. Perhaps the most important is that, especially for flows,

[9] These default declining-balances rates were initially used by Hulten and Wykoff (1981, p.94). They report that for the four NIPA equipment categories for which they had data on used asset prices, the average declining-balance rate was estimated to be 1.65. Similarly, the average declining-balance rate for the two NIPA structures categories for which they had data was estimated to be 0.91.

[10] Sliker (2012) has shown that the age-price profile for an entire cohort of assets may be geometric even when the age-price profiles for individual assets aren't, provided that the associated services lives are taken from a gamma density function. This result does not hold when the declining balance rate is less than one. In that case, individual age-price profiles increase with age and most retirements occur at a very early age.

it eliminates the substitution bias in measures of growth that are derived using fixed-weighted indexes. This bias tends to cause an understatement of growth for periods before the reference year and an overstatement of growth for periods after the reference year. Chain-type indexes produce the most accurate estimates of the growth rate from a given year to the next because they use the most relevant weights, that is, weights that reflect the composition of these two years. When a chain-type index is used, the levels of the estimates change when the reference period is changed, but the growth rates of the various series do not change. Consequently, the use of a chain-type index avoids the “rewriting of economic history” that occurs when the reference period of a fixed-weighted index is updated.[11] Because the Fisher index formula treats both time periods being compared symmetrically, Fisher chain-type indexes are likely to yield results that are more acceptable in the presence of fluctuations.

There are two properties that hold for aggregates when fixed weights are used that do not hold when chained weights are used. The first is that over the service life of a group of assets, depreciation charges on them will sum up to their value when they were initially purchased. The second is the stock-flow identity. In other words, with chain-type indexes, it is no longer the case that the beginning-of-year value of the net stock plus gross investment less depreciation equals the end-of-year value of the net stock.

Standard methods of chain-type Fisher aggregation cannot be used for the aggregation of real net investment. Current-dollar net investment sometimes takes on negative values. Because it is impossible to take the square root of a negative number, BEA found it necessary to measure real net investment using other techniques. Specifically, BEA measures real net investment by subtracting chained-dollar depreciation from chained-dollar gross investment.

Users also need to exercise caution when using chained-dollar estimates in periods far from the reference year. While the long-term growth rates of chained-dollar aggregates can be usefully compared, comparisons of the chained-dollar levels of two different types of assets should not be made. In periods far from the reference year there are instances where the chained-dollar value of a component exceeds the chained-dollar value of the aggregate of which it is part. These problems are particularly evident when the aggregate contains goods, such as computer equipment and software, whose rates of price inflation are very different from the rates for other goods. These problems affect stock estimates less than depreciation estimates

[11] Chained-dollar aggregates are not additive and should not be used for comparing one aggregate to another at a point in time. Such comparisons should be made using aggregates based on the current-cost valuation, which are additive.

because these goods comprise a smaller share of gross investment and wealth stocks than they do of depreciation.

5.0 Capital Services

Nonfinancial fixed assets yield services continuously over their service lives. These services are often called **capital input**, particularly when dealing with production functions. Thus, assets such as houses, cars, pianos, and cranes yield services that people often pay rent to obtain. By the opportunity cost principle, if the owners of these assets decide to forego this rent and use the assets themselves, they must place a value on their services that is greater than this rent. Even when assets cannot be rented out in practice, the principles of optimization over time can be used to obtain the same implicit rental value.

This brings us to the **fundamental equation of capital theory**. This equation, which has been known for more than a hundred years, states that in equilibrium the price of a capital asset will equal the discounted present value of the net income expected to be derived from owning it over its lifetime.[12] For nonfinancial assets that are used by their owner, the net income is equal to the implicit rental value or **user cost** of the asset's services. In other words, this net income is given by the asset's gross income, less any associated inputs such as maintenance and repairs, fuel, etc. that we can describe as being operating costs. In the case of housing, think of operating costs as being any costs that a landlord might incur.

This fundamental equation can be used to derive the user cost of capital measure of an asset's services. This is done in appendix A of this primer. The standard user cost measure expresses the expected nominal value of the asset’s services as the sum of three components: the expected nominal net return to capital, the expected decline in the price of the asset during the year, and the expected value of operating expenses (which, for simplicity, we shall assume are equal to zero in our examples). The expected net return to capital is measured by foregone interest, i.e., as the product of the price of the asset at the beginning year and the rate of return that could be earned if the funds tied up in the asset were invested elsewhere. A mathematical expression of the standard user cost measure is given in equation (1) below.

[12] The equation has been known at least since the time of Böhm-Bawerk (1891).

(1) $C^e_{s,t} = r^e_t P_{s,t} + (P_{s,t} - P^e_{s+1,t+1}) + O^e_{s,t}$

where $P_{s,t}$ denotes the purchase price of an s-year old asset at the beginning of year t; $P^e_{s+1,t+1}$ denotes its expected purchase price at the beginning of year t+1 when the asset is one year older; $C^e_{s,t}$ denotes the expected value of the services of this s-year old asset in year t; $O^e_{s,t}$ denotes the expected operating expenses for this s-year old asset in year t; and r^e_t denotes the expected nominal discount rate (i.e., the rate of return on the best alternative investment) in year t.[13]

There are two basic ways to estimate capital services. The first makes use of assumptions about each asset's age-price profile. The second makes use of assumptions about each asset's age-efficiency profile, a different but related concept. The following exposition will explain the basics of the two methods and then show how the two are related to each other.

5.1 Estimation of Capital Services Using Data Pertaining to Wealth Stocks

The first method for estimating capital services makes use of assumptions about each asset's age-price profile. It, therefore, can be implemented in a manner that is entirely consistent with the methodology used by BEA to estimate capital stocks and depreciation. To illustrate this methodology, we will first compute an example using straight-line depreciation and will then repeat the exercise using an example based on geometric depreciation.

5.1.1 Example Using Straight-line Depreciation

Let us assume that the actual prices observed for various ages of our asset at the beginning of year t-1 are as shown in the first column of table 8 and that the prices that are expected to be in effect at the beginning of year t are as shown in the second column of this table. Let us further assume that the expected nominal rate of return on the asset is .092 in both years.

[13] In this formulation, every year a new set of expectations of the discount rate and rate of inflation that will be in effect in specified future years is formed. These may be different from the expectations that were held in prior years. However, the expected service life is treated as a known fact so that expectations of it are not permitted to change over time.

Table 8 - Actual and Expected Prices of the Asset as of the Beginning of the Year, by Age of Asset

Year	t-1	t
Age of asset		
0-years old (new)	100	104
1-year old	75	78
2-years old	50	52
3-years old	25	26
Nominal expected rate of return	.092	.092

Also assume that in each of the two years one unit of the asset is produced and installed so that in each year the stock consists of one unit of the asset of every possible age. Then, we can compute the (expected) net return to capital and the decline in the market value of the asset for each unit in the stock as in table 9. Summing these, we obtain a value of 117 for the services of the stock in year t-1 using the user-cost measure of capital services.

Table 9 - Computation of Service Values, by Age of Asset in Year t-1

Age of asset	Net return to capital in year t-1 (1)	Decline in market value during year t-1 (2)	User cost (1)+(2)
0 years-old	.092 * 100 = 9.2	100 - 78 = 22	31.2
1-year old	.092 * 75 = 6.9	75 - 52 = 23	29.9
2-years old	.092 * 50 = 4.6	50 - 26 = 24	28.6
3-years old	.092 * 25 = 2.3	25 - 0 = 25	27.3
Total stock	9.2 + 6.9 + 4.6 + 2.3 = 23	22 + 23+ 24 + 25 = 94	117

5.1.2 Example Using Geometric Depreciation

We now repeat the calculation of the previous section using an example based on geometric depreciation. Let us assume that the actual prices observed for various ages of our asset at the beginning of year t-1 are as shown in the first column of table 10 and that the prices that are expected to be in effect at the beginning of year t are as shown in the second column of this table.

Table 10 - Actual and Expected Prices of the Asset as of the Beginning of the Year, by Age of Asset

Year	t-1	t
Age		
0-years old (new)	100	104
1-year old	60	62.4
2-years old	36	37.44
3-years old	21.6	22.46
…………………….	…….	……………
10-years old	0.60	0.63
Nominal expected rate of return	.092	.092

Let us further assume that the expected nominal rate of return on the asset is .092 in both years. Also assume that in each of the two years one unit of the asset is produced and installed so that in each year the stock consists of one unit of the asset of every possible age. Then, we can compute the (expected) net return to capital and the decline in the market value of the asset for each unit in the stock as in table 11. Summing these we obtain a value 117 for the services of the stock in year t-1 using the user cost measure of capital services. By a remarkable coincidence, this is precisely equal to the total obtained in the previous section using straight-line depreciation.

Table 11 - Computation of Service Values, by Age of Asset, in Year t-1

Age of asset	Net return to capital in year t-1 (1)	Decline in market value during year t-1 (2)	User cost = (1)+(2) (3)
0 years-old	.092 * 100 = 9.2	100 - 62.4 = 37.6	46.8
1-year old	.092 * 60 = 5.52	60 - 37.44 = 22.56	28.08
2-years old	.092 * 36 = 3.31	36 - 22.46 = 13.54	16.85
3-years old	.092 * 21.6 = 1.99	21.6 -13.48 = 8.12	10.11
…………….	………………………..	……………………………..	……….
10-years old	.092 * 0.60 = 0.06	0.61 - 0.38 = 0.23	0.28
…………….	………………………..	……………………………..	……….
Total stock[14]	9.2 + 5.52 + 3.31 + 1.99 + +0.06 + ... = 23	37.6 + 22.56 + 13.54 + 8.12 + ... + 0.23 + ... = 94	46.8 + 28.08 + 16.85 + 10.11+ ... +0.28 + ... = 117

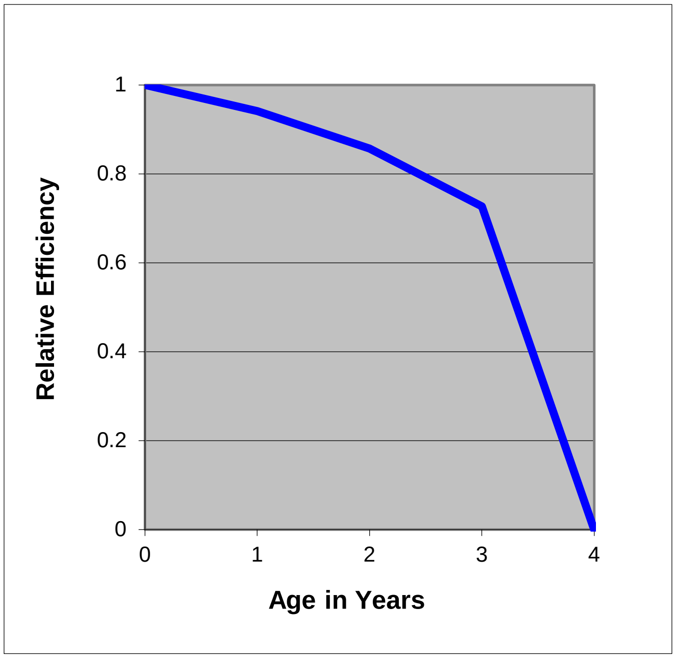
5.2 Age-efficiency Profiles

Parallel to the concept of age-price profiles, there is a concept of age-efficiency profiles that is used in estimating capital input, i.e., capital services. Specifically, an asset's **age-efficiency profile** measures the value of the services of an asset at various ages relative to the value of the services it produced when new, all services being measured in terms of rental prices of the reference year.[15] The profile generally declines over the asset's life as shown in figure 7. Each point on the curve is described as denoting a relative efficiency of capital. These are commonly thought of as being relative marginal products.

[14] The sums of the terms of the terms in each of the three columns are derived using the formula for the sum of an infinite geometric progression. Thus, the sum for each column is given by dividing the first term of the sum by 1 - .6, as each term in the progression is equal to 60 percent of the value of the prior term.

[15] The rental price of an asset is the rental value of a new unit of that type of asset. Thus, relative efficiencies are quantity measures and not prices.

Figure 7 - A Typical Age-efficiency Profile



Just as there is an "apparent" age-price profile, there is also an "apparent" age-efficiency profile. This latter profile shows, for a given type of asset, the relative efficiencies for assets of different ages that exist at a given point in time. Thus, it gives the relative rental values of assets that differ solely with respect to their age.

Data from the age-efficiency profile can be used to construct a stock that is proportionate to the services that are produced. Specifically, the constant-price **productive stock** is defined to be equal to the un-depreciated (gross) value of the units of the asset in the stock, measured in constant prices, weighted by the appropriate relative efficiencies of capital. It measures the stock solely in terms of its ability to contribute to production in the current year. Likewise, one can define the constant-price value of the wealth (net) stock as being the un-depreciated value of the units of the asset in the stock, measured in constant prices, weighted by the appropriate values from the age-price profile. It measures the stock in terms of its ability to contribute to production in the current and all future years.

5.3 Estimation of Service Values Using Data Pertaining to Productive Stocks

The capital stock's services can also be estimated using data pertaining to age-efficiency profiles and the productive stock. This is the approach that BLS takes in estimating these services in its work on multifactor productivity.

5.3.1 Estimates Using Straight-line Depreciation

In table 8 of section 5.1.1, we assumed that the rate of inflation in the asset's price was expected to be 4 percent in year t-1 and that the asset's rate of return was expected to be 9.2 percent in years t-1 and t. Let us extend this example by assuming that the rate of return is also expected to be 9.2 percent in years t+1 and t+2 and that the rate of inflation in the asset's price is also expected to be 4 percent in years t, t+1, and t+2. In appendix B of this primer, equation B6 gives a general expression for the fundamental equation of capital theory. Substituting the data given above into this equation, we obtain:

$$(2) \qquad P_{0,t-1} = C^{e}_{0,t-1} \left(\frac{\phi_0}{(1+.092)} + \frac{\phi_1 * 1.04}{(1+.092)^2} + \frac{\phi_2 * (1.04)^2}{(1+.092)^3} + \frac{\phi_3 * (1.04)^3}{(1+.092)^4} \right)$$

where $P_{0,t-1}$ is the price of a new asset at the beginning of year t-1,

$C^{e}_{0,t-1}$ is the expected service value of a new asset in year t-1, and

ϕ_s is the relative efficiency of an s-year old asset.

Let us further assume that the relative efficiencies during the four years of the asset's service life are 1, .958, .917, and .875. Substituting these values and the asset's initial purchase price of 100 into equation (2), we obtain

$$(3) \qquad 100 = C^{e}_{0,t-1} \left(\frac{1}{(1+.092)} + \frac{.958 * 1.04}{(1+.092)^2} + \frac{.917 * (1.04)^2}{(1+.092)^3} + \frac{.875 * (1.04)^3}{(1+.092)^4} \right)$$

Solving equation (3), we obtain a value of 31.2 for $C^{e}_{0,t-1}$.

The services of each of the four assets in the stock can then be estimated by multiplying the services of a new asset (31.2) by the respective relative efficiencies so that we obtain the following expression for the services of the stock

$$C^{e}_{0,t-1} + C^{e}_{1,t-1} + C^{e}_{2,t-1} + C^{e}_{3,t-1} = C^{e}_{0,t-1} \, (1 + .958 + .917 + .875)$$

$$= C^{e}_{0,t-1} * 3.75 = 31.2 * 3.75 = 117$$

The value of the services of the stock is estimated to be 117, which is precisely the same value that was obtained in section 5.1.1 when estimating services values from wealth stocks. This is not just an extraordinary coincidence. The values assumed for the example were chosen to obtain this result. Given the assumed nominal rate of return and rate of inflation in our example, the age-price profile that was assumed when we constructed the depreciation and wealth stock estimates implies an apparent age-efficiency profile that is identical to the actual age-efficiency profile that was assumed when we estimated the value of capital services using the second method.

5.3.2 Estimates Using Geometric Depreciation

We can repeat the computations of the preceding section for our example using geometric depreciation. Just as we extended table 8, we extend table 10 by assuming that the rate of return is expected to be 9.2 percent in year t-1 and all subsequent years and that the rate of inflation in the asset's price is expected to be 4 percent in year t and in all subsequent years. Substituting the data given above into equation B6, we obtain:

(4) $$P_{0,t-1} = C^{e}_{0,t-1} \left(\frac{\phi_0}{(1+.092)} + \frac{\phi_1 * 1.04}{(1+.092)^2} + \frac{\phi_2 * (1.04)^2}{(1+.092)^3} + \ldots + \ldots \frac{\phi_n * (1.04)^n}{(1+.092)^{n+1}} + \ldots \right)$$

where $P_{0,t-1}$ is the price of a new asset at the beginning of year t-1,

$C^{e}_{0,t-1}$ is the expected service value of a new asset in year t-1, and

ϕ_s is the relative efficiency of an s-year old asset.

Let us further assume that the relative efficiencies decline at a rate of 40 percent per annum so that they are given by the following sequence 1, 0.6, 0.36, 0.216, etc. Substituting these values and the asset's initial purchase price of 100 into equation (4), we obtain

(5)

$$100 = C^{e}_{0,t-1} \left(\frac{1}{(1+.092)} + \frac{0.6*1.04}{(1+.092)^2} + \frac{0.36*(1.04)^2}{(1+.092)^3} + \ldots + \ldots \frac{(0.6)^n*(1.04)^n}{(1+.092)^{n+1}} + \ldots \right)$$

Solving equation (5), we obtain a value of 46.8 for $C^{e}_{0,t-1}$.

The services of each of the assets in the stock can then be estimated by multiplying the services of a new asset (46.8) by the respective relative efficiencies so that we obtain the following expression for the services of the stock

$$C^{e}_{0,t-1} + C^{e}_{1,t-1} + \ldots + C^{e}_{n,t-1} + \ldots = C^{e}_{0,t-1} \; (1 + .6 + \ldots + (.6)^n + \ldots)$$

$$= C^{e}_{0,t-1} * 2.5 = 46.8 * 2.5 = 117$$

The value of the services of the stock is estimated to be 117, which is the same value that was obtained in section 5.1.2 when estimating services values from wealth stocks. Once again this is not a coincidence as the values assumed for the example were chosen to obtain this result. Our assumed age-price profile has prices that decline at a constant rate of 40 percent per annum. It implies an age-efficiency profile that also declines at a constant rate of 40 percent per annum. The fact that the age-efficiency and age-price profiles are identical is a property specific to strictly geometric depreciation and is independent of the value of the real rate of return.

5.4 Wealth Versus Productive Stocks

The difference between wealth and productive stocks is more readily seen by reference to the examples we have been following. Let us first trace things through by continuing the example given earlier based on straight-line depreciation. As shown in table 8, in year t-1 the stock consists of four units of the asset: one new unit of the asset, one 1-year old asset, one 2-year old asset, and one 3-year old asset. As of the beginning of that year, the market value of this stock is given by 100 plus 75 plus 50 plus 25 or 250. This is what we have termed the wealth stock. From the example, we know that this stock yields services worth 117 in year t-1.

Now suppose that we had a hypothetical (alternative) stock that consisted of only 3.75 (i.e., 1 plus .958 plus .917 plus .875) units of a new asset of the given type. Multiplying 3.75 by the user cost of a new asset found in in equation (3), 31.2, we know that this hypothetical stock would yield services worth 117 in year t-1, the same as the actual stock does. The market or wealth value of the hypothetical stock would be 375, which is obtained by multiplying 3.75 by the price of a new asset (100). This is considerably higher than wealth value of our actual stock (250). The reason for this higher value is that in the future the hypothetical stock would yield greater services than would the actual stock as all assets in it will yield services for another 3 years. This hypothetical stock is what Harper (1982) and many others have termed the "productive stock" of the asset. Constant-price capital services are proportionate to this productive stock. But, as we have just seen, identical values of capital services can also be estimated from wealth stocks when the appropriate assumptions are made.

We can repeat these calculations of the values of wealth and productive stocks using our example based on geometric depreciation. In this example, one new unit of the asset entered the stock at the beginning of year t-1 as well as at the beginning of all prior years. Consequently, in year t-1 the stock consists of one new unit of the asset, one 1-year old asset, one 2-year old asset, one 3-year old asset, etc., i.e., one unit of every age. As of the beginning of that year, the market value of this stock is given by 100 plus 60 plus 36, etc. Using the formula for the sum of the terms of a geometric progression, we find that these sum to a value of 100/(1-.6) = 250. This is what we have termed the wealth stock. From the example, we know that this stock yields services worth 117 in year t- 1.

Now suppose that we had a stock that consisted of 1 plus .6 plus .36, etc. equals 2.5 units of a new asset of the given type. This hypothetical stock is the productive stock. Multiplying 2.5 by the user cost of a new asset, 46.8 (from section 5.3.2), we know that this stock would also yield services worth 117 in year t-1. The market value of this productive stock would be equal to 100 times 2.5 or 250, which is precisely equal to the value of the actual wealth stock.

The result that the market value of the productive (hypothetical) stock is equal to the market value of the actual stock is specific to strictly geometric depreciation. When depreciation is based on any other age-price profile, the market value of the productive stock is not equal to the market value of the wealth stock. The convenience of having the market values of these two stocks be equal to each other is an important reason why many theorists have used geometric depreciation in their work.

Geometric depreciation has another important property when estimates are expressed in constant (fixed) prices of a given base year. With this valuation method, economic depreciation is equal to the amount of gross investment needed to maintain the value of net stock of capital intact. In other words, if gross investment equals the value of economic depreciation, the value of the capital stock at the end of the year will be equal to its value at the beginning of the year. Similarly, with constant prices, economic replacement is equal to the amount of gross investment needed to maintain the value of the productive stock intact. When depreciation is strictly geometric, the value of economic depreciation equals the value of economic replacement. With non-geometric depreciation, economic depreciation does not equal economic replacement. Having the two concepts be equal in value to each is a property that is especially appealing to economic theorists.

5.5 The Relationship Between Age-price and Age-efficiency Profiles

In section 5.1, given a set of assumptions about future nominal interest rates, rates of inflation in the asset's price, and the asset's age-price profile, we were able to estimate the value of the asset's services. In section 5.3, given the same set of assumptions about future interest rates and rates of inflation in the asset's price, we saw that by making a particular assumption about the asset's age-efficiency profile, we were also able to estimate the value of the asset's services. Both methodologies yielded identical estimates. This suggests that the age-price profiles used in sections 5.1.1 and 5.1.2, respectively, imply the age-efficiency profiles used in sections 5.3.1 and 5.3.2 and vice versa.

The fact that this is precisely what is happening can be shown by re-examining equation (2). For a given set of future nominal interest rates, rates of inflation, and relative efficiencies of capital, a given service value of a new asset is consistent with one and only one value of the price of the new asset. Appendix B shows that there is an entire system of equations similar to equation (2), one for each possible age of the asset. Solving all of these equations one at a time will completely determine the apparent age-price profile for the asset, the profile that is in effect for the given year.

A number of different shaped age-efficiency profiles are used in empirical work, and it useful to examine what their implied age-price profiles look like, and how the profiles are affected by differences in the assumed rate of return. Figure 8 shows four patterns of age-efficiency profiles for an asset with a service life of ten years: one exhibits no declines in efficiency (often termed the "one hoss shay" pattern), a second exhibits hyperbolic declines in efficiency that are generated by a beta decay function with a beta parameter of .75, a third also exhibits hyperbolic declines in efficiency but is based on a beta parameter of .5, while the fourth exhibits relative efficiencies that decay geometrically at a constant rate of 16.5 percent per annum.[16]

Figure 8- Assumed Age-efficiency Profiles

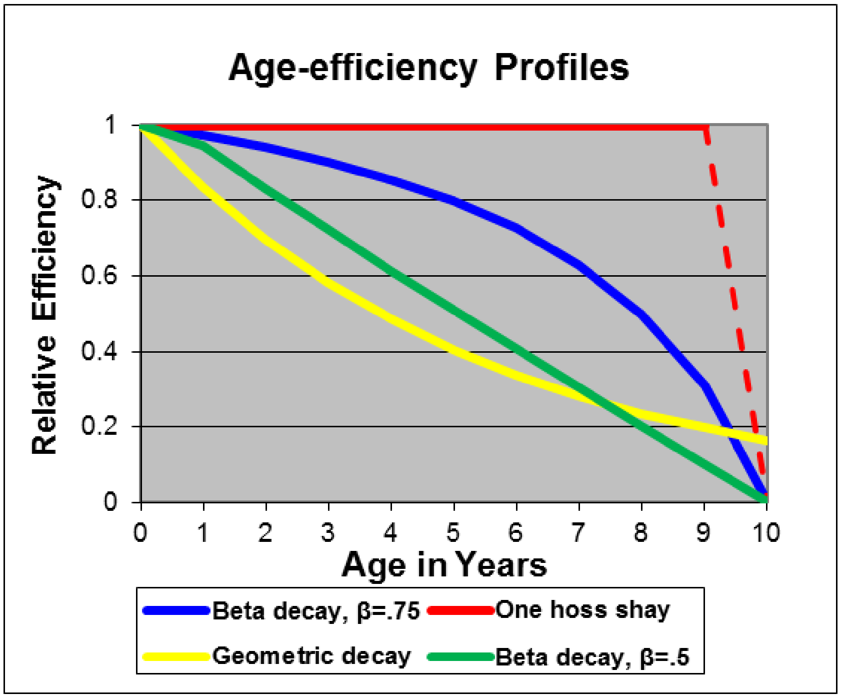


Figure 9 shows the age-price profiles that are consistent with the profiles from figure 8 when it is expected that the real rate of return will be 0 percent in all future periods[17]. Of note, the one-hoss shay pattern yields an age-price profile that declines linearly to zero in a straight-line manner. The pattern of geometric declines in relative efficiency yields an age-price profile that declines at the same constant geometric rate as do the relative efficiencies.[18] The age-price profiles

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[16] The beta decay function is given by (L - s) / (L - β s) where L is the service life, s is the age of the asset, and β is a constant. Initially BLS used a profile with beta equal to 0.9 to estimate services for structures and one with beta equal to 0.75 to estimate services for equipment. BLS now uses a profile with beta equal to 0.75 to estimate services for structures and one with beta equal to 0.5 to estimate services for equipment. It is important to note that this profile is for a single asset. Because BLS uses an asset discard distribution, the mean age-price and age-efficiency profiles for an entire cohort of investment in an asset are closer to the geometric pattern than are the patterns shown in the charts here.

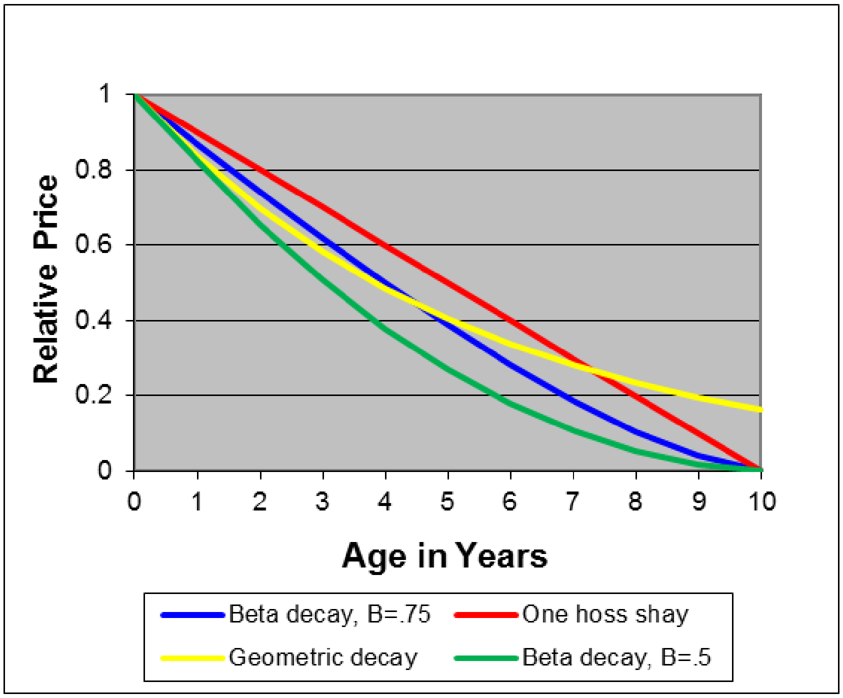
[17] The expected real own rate of return in year t , ρ^e_t, is defined by the equation $1+\rho^e_t{}^* = (1+r^e_t)/(1+\dot{P}^e_{0,t})$

where i^e_t is the expected nominal rate of return on the asset in year t and $\dot{P}^e_{0,t}$ is the expected rate of

inflation in the asset's price in year t.) This is often approximated by $r^e_t - \dot{P}^e_{0,t}$.

[18] This result obtains because the example uses a pattern that is "strictly geometric," which is the pattern predominantly used by BEA and throughout the economics literature. Here, technically, the patterns are based on a life that is infinite. Note that the asset has a positive value at the end of its supposed service life

Figure 9 – Associated Age-price Profiles Given a 0-Percent Real Rate of Return



that are implied by the beta-decay age-efficiency profiles lie between those implied by the one-hoss shay and geometric-decay patterns. Those age-efficiency profiles that exhibit the fastest declines in relative efficiency imply age-price profiles with the fastest declines in prices.

Figure 10 shows the age-price profiles that are implied by the age-efficiency profiles from figure 9 when it is expected that the real rate of return will be 4 percent in all future periods. Compared to the age-price profiles of figure 9, the corresponding profiles in figure 10 exhibit slower rates of price decline. Thus, for example, the one-hoss shay pattern's implied age-price profile is now concave to the origin rather than a straight-line. The effects of using different real interest rates are shown more clearly in figure 11.

of 10 years. Also, the constant rate depreciation rate of 16.5 percent that is obtained is the "default" rate that BEA would assign to an asset with a service life of 10 years when no empirical information was available regarding its actual depreciation rate.

Figure 10 - Associated Age-price Profiles Given a 4-Percent Real Rate of Return

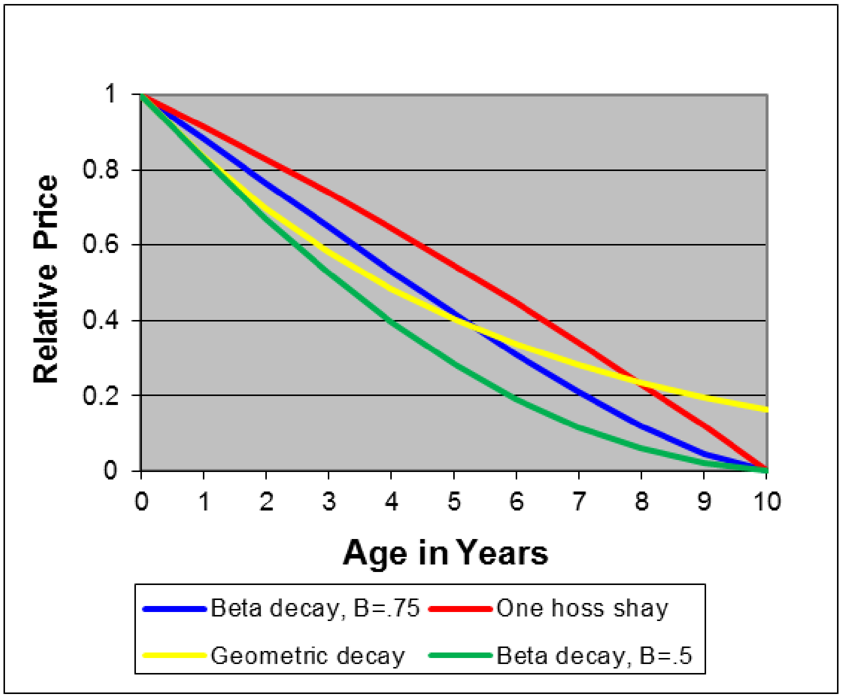


Figure 11 - Associated Age-price Profiles at 0 and 4 Percent Real Rates of Return

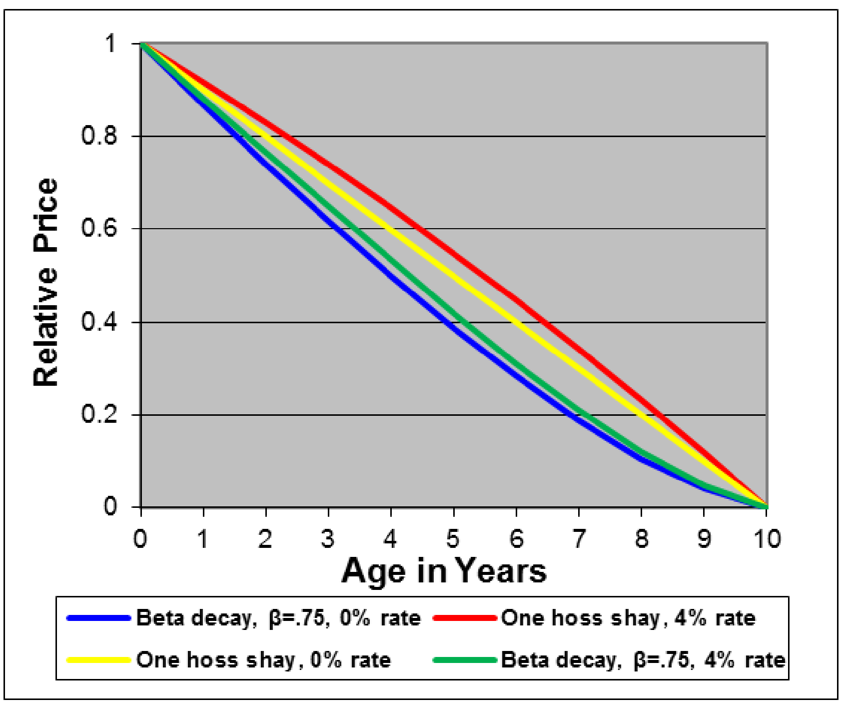
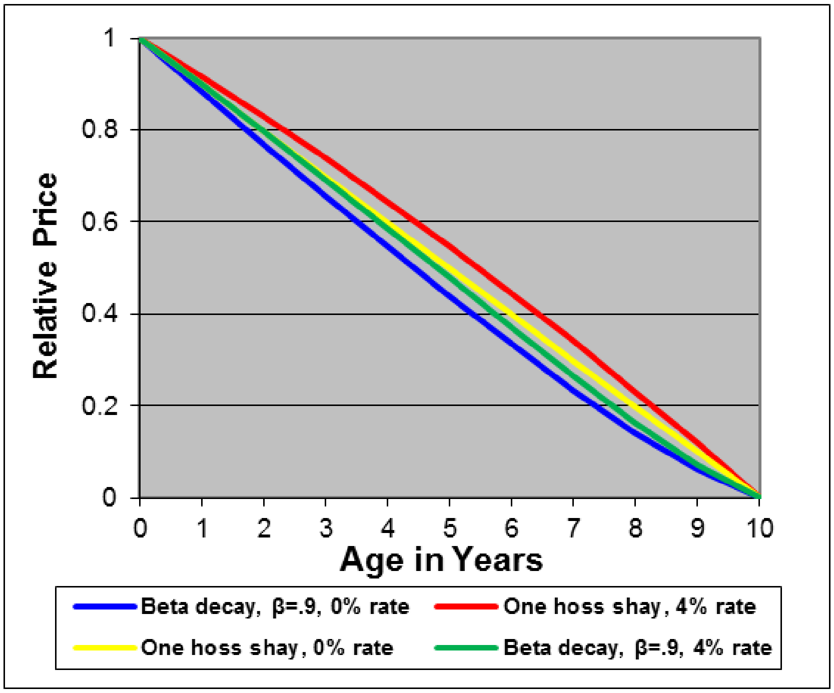


Figure 12 differs from figure 11 only in that we set beta equal to 0.9, the parameter that BLS used for structures in its initial estimates

Figure 12 - Associated Age-price Profiles at 0 and 4 Percent Real Rates of Return



made in the 1980's.[19] Here, we see that with the 4-percent real rate of return, the implied age-price profile for the beta decay age-efficiency pattern (the green line) is very close to the straight-line pattern (the yellow line).

There is one notable exception to these results. The age-price profiles resulting from strictly geometric-decay are identical in both figures 9 and 10. This is, in fact, a general result. With geometric decay, the implied age-price profile is always equal to the age-efficiency profile regardless of what the real interest rate is. In fact, it is possible to prove that the geometric pattern is the only exception to these principles. In other words, if an asset's age-price profile is held constant over time, then its implied age-efficiency profile will vary over time with changes in the real own interest rate unless the age-price profile exhibits a pattern of strictly geometric declines in value. Conversely, if an asset's age-efficiency profile is held constant over time, then its implied age-price profile will vary over time with changes in the real own interest rate unless the age-efficiency profile exhibits a pattern of strictly geometric declines in relative efficiency.[20]

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[19] Jack Faucett Associates recommended that BLS use a beta equal to 0.9 in estimating capital stocks for input-output sectors, see BLS (1970). BLS used this parameter for structures in its stock estimates for input-output industries, see BLS (1979). However, the productivity division of BLS uses a beta of .75 for structures, see BLS (1983).

[20] For a proof of this see Appendix C of this primer.

5.6 Aggregation of Capital Services

In section 4.5 of this primer we discussed how estimates of net stocks and depreciation could be produced for aggregates consisting of more than one type of asset. The methodology involved the use of Fisher chain-price indexes. Capital services can be aggregated in an analogous manner. To aggregate capital services, however, we must chain together growth rates of rental prices rather than growth rates of purchase prices. This implies that changes in real own interest rates are treated like changes in prices. In the measurement of real economic income and capital services, the question of how changes in interest rates should be treated remains unsettled, and the treatment stated explained above (which treats them as price changes) has not received universal acceptance.[21] Nevertheless, it is an essential part of the capital accounting framework developed by Dale Jorgenson and various co-authors.

6.0 Integration of Capital Stocks and Services

The ultimate goal of capital stock estimation is the development of an integrated system of stocks and flows, i.e., a system in which estimates of capital stocks, depreciation, and capital services are estimated using a common set of assumptions and concepts. Practically speaking, how would the methodology that BEA uses to estimate capital stocks and depreciation and the methodology that BLS uses to estimate capital services have to be modified in order to have an integrated system that would be theoretically satisfying?

At this point we need to clarify some subtleties that were glossed over earlier in this primer. With some exceptions that were noted earlier, BEA generally assumes that the age-price profile for a specific type of asset is the same for all vintages of the asset. Consequently, the actual age-price profile that is used to estimate depreciation and capital stocks is generally equal to the apparent age-price profile in every year. This holds true for both current and constant-price estimates. Similarly, BLS assumes that the age-efficiency profile for a specific type of asset is the same for all vintages of the asset. Thus, the actual age-efficiency profile that is used to estimate capital services is equal to the apparent age-efficiency profile in every year.

The biggest conceptual challenge to capital stock and flow integration results from the fact that changes in real interest rates will generally have different impacts on capital stocks and flows (by having different impacts on the wealth and productive stocks). These

[21] For example, see Hicks (1965, pp. 174-188).

difficulties emerge in the estimation of nominal measures; integration of real estimates is less of a problem.

6.1 Integration in Constant Prices

An integrated system of capital stocks and flows (services) that is consistent with our theoretical constructs can be developed in constant prices in a straight-forward manner. By definition, in a constant-price system, all prices are held constant at their values in the reference year. Which prices are we talking about – prices of assets or the rental prices of these asset's services? The answer is both. We want both the apparent age-price profiles that underlie the stock estimates and the apparent age-efficiency profiles that underlie the service estimates to remain fixed over time at their values in the reference year.

Such estimates can be made starting with either fixed asset prices or fixed rental prices using the methods developed earlier in this primer. We have seen that the fundamental equation of capital theory is a series of equations that shows the relationship between three sets of variables: (1) the age-efficiency profile of the asset, (2) the expected real own rate of return (which is a function of the expected nominal rate of return of the asset and the expected rate of inflation in the price of the asset) and (3) the implied age-price profile for the asset. If we know (1) and (2), we can solve for (3). Conversely, if we know (2) and (3), we can solve for (1).

We can achieve stock-flow integration in constant prices because, by definition, in a constant-price system we hold the real own rate of return constant. Then, if we hold the asset's age-price profile constant at its value in the reference year, the implied age-efficiency profile will also be constant at its value in the reference year. We can achieve the same result starting with age-efficiency profiles. If we hold the asset's age-efficiency profile constant at its values in the reference year, then because the real own rate of return is also held constant, the asset's implied age-price profile will also be constant at its value in the reference year.

We must recognize, however, that BEA's geometric age-price profiles imply geometric age-efficiency profiles that are different from the hyperbolic profiles assumed by BLS. Likewise, the BLS hyperbolic age-efficiency profiles imply age-price profiles that are different from the geometric profiles used by BEA. Thus, there does not appear to be a way to integrate BLS estimates of capital services with BEA estimates of net stocks and depreciation in a manner that meets all of our ideals regarding internal consistency.[22]

[22] Recent research by Sliker (2012) implies that if individual assets all had hyperbolic age price profiles, the stock as a whole could have geometric depreciation if the asset discard distribution was right skewed with a

6.2 Integration in Current Prices

Because an asset's age-efficiency profile and its age-price profile are not totally independent of each other, any methodology for measuring capital stocks and flows in current prices cannot treat them as if they were independent of each other. This implies that integration can be achieved in either of three ways: (1) the age-price profile is held constant over time, (2) the age-efficiency profile is held constant over time, or (3) geometric depreciation is assumed. In general, the goal of having a perfectly integrated system of capital stocks and flows is not attainable in current-prices. In current price measurement, an asset's expected nominal rate of return and its expected rate of inflation can vary from year to year. Because the real own rate of return is not held constant, then if the asset's age-price profile is held constant, its implied age-efficiency profile cannot be constant. Conversely, because the real own rate of return is not held constant, then if the asset's age-efficiency profile is held constant, its implied age-price profile cannot be constant.

There is an important exception to the above conclusion. A perfectly integrated system of capital stocks and services can be obtained in current prices when all age-efficiency profiles and age-price profiles decline at strictly (constant) geometric rates. As noted earlier, an age-price profile that declines at a constant rate of r-percent per annum implies an age-efficiency profile that also declines at a constant rate of r-percent per annum, regardless of what the real own interest rate and the rate of inflation in the price of the asset are.[23]

It is important to recognize that the results **do not** show that we cannot obtain current-price estimates of capital stocks and flows that are based on a consistent set of assumptions and that, therefore, form an integrated system. They do show, however, that the price of obtaining such integration is that either the age-price or the age-efficiency profile must be permitted to vary over time.

There is an additional aspect to this that has not received any attention in the literature. If we permit the age-efficiency profile of a given type of asset to vary over time, then current-price estimates of

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very long tail. However, BLS uses a normal distribution that is symmetric and that has had its tails truncated.

[23] Note that an integrated system of stocks and flows can also be constructed if one adopts the expedient of holding real own interest rates constant. With such a system one can obtain practical measures of capital services at a cost of having estimates that do little to explain many real-world phenomena during periods, such as 1979-85, when there were obviously large changes in real own interest rates. Katz (2009) reports that a Eurostat task force on which he served recommended that, for the purpose of estimating GDP in the Eastern European countries that were acceding to join the European Union in 2004, owner-occupied dwelling services should be estimated by a user cost of capital measure in which the real rate of interest was held constant. This avoided the sharp volatility that often plagues empirical estimates based on more realistic assumptions.

this type of asset's services would involve chaining growth rates over different vintages of the asset. Conversely, if we permit the age-price profile of a given type of asset to vary over time, then current-price estimates of this type of asset's net stock or depreciation would involve chaining growth rates over different vintages of the asset. Currently, neither BEA nor BLS produce estimates that involve chaining growth rates over different vintages of an asset.

6.3 Summary and Conclusion

The ideal of having a perfectly integrated system of capital stocks and flows that are based on a common set of set of assumptions and concepts is difficult to achieve. We have shown how it can be achieved in constant prices. But, this goal cannot be achieved in current prices unless all age-price and age-efficiency profiles are strictly geometric or unless either the age-price profile or the age-efficiency profile is permitted to vary over time. Because BEA uses geometric age-price profiles for most assets and BLS assumes age-efficiency profiles that are far from the geometric shape, it is not possible to perfectly integrate BEA estimates of capital stocks and depreciation with BLS estimates of capital services.

7.0 General Summary and Conclusion

As of 2012, BEA estimates the net stocks and depreciation of all assets other than autos using the perpetual inventory method. The estimated values are essentially weighted averages of past investment in the assets. Each asset's age-price profile is held constant over time in both current and constant prices. As a result, in constant prices, depreciation over the service life of each asset equals the asset's initial purchase price. This property is the cornerstone of BEA's methodology.

BLS estimates productive stocks and capital services using a similar method in which these values are also weighted averages of past investment. Each asset's age-efficiency profile is held constant over time in both current and constant prices. Identical constant-price estimates of capital services can be obtained from estimates of net stocks and depreciation without making use of the concept of productive stocks.

An asset's age-price profile and its age-efficiency profile are related to each other through the fundamental equation of capital theory. For any given real own interest rate, the data from the asset's age-price profile can be used to derive the implied age-efficiency profile. Conversely, for any given real own interest rate, the data from the asset's age-efficiency profile can be used to derive the implied age-price profile. It is impossible for both profiles to be constant over time unless the real own interest rate is constant over time or unless both profiles are strictly geometric. Because BLS assumes age-efficiency profiles that are not geometric, it does not appear possible to perfectly integrate BLS estimates of capital services with BEA estimates of capital stocks and depreciation.

Appendix A - Derivation of the User Cost of Capital Measure of the Implicit Rental Value of an Asset's Services

The user cost of capital measure of the implicit rental value of an asset's services is directly derived from the fundamental equation of capital theory. This equation states that, in equilibrium, the price of a capital asset will equal the discounted present value of the net income expected to be derived from owning it over its lifetime. For a durable good that is used by its owner, the net income is given by the implicit rental value or user cost of capital for the asset, i.e., its gross income, less any associated inputs such as maintenance and repairs, fuel, etc. that we can describe as being operating costs.

To spell out the equation concretely, let $P_{s,t}$ denote the purchase price of an s-year old asset at the beginning of year t; $P^e_{s+1,t+1}$ denote its expected purchase price at the beginning of year t+1 when the asset is one year older; $C^e_{s,t}$ denote the expected value of the services of this s-year old asset in year t; $O^e_{s,t}$ denote the expected operating expenses for this s-year old asset in year t; and r^e_t denote the expected nominal discount rate (i.e., the rate of return on the best alternative investment) in year t. (We assume that all of the assets are of type i so that there is no need to include a subscript for this variable.) Expected variables are measured as of the beginning of year t. Assume that the entire value of the asset's services in any year will be received at the end of the year, and that the asset is expected to have a service life of m years. From the definition of discounted present value, the fundamental equation is given by

$$\text{(A1)}\quad \begin{aligned} P_{s,t} = {} & \frac{C^e_{s,t}}{1+r^e_t} + \frac{C^e_{s+1,t+1}}{(1+r^e_t)(1+r^e_{t+1})} + \ldots + \frac{C^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t}(1+r^e_j)} \\ & - \frac{O^e_{s,t}}{1+r^e_t} - \frac{O^e_{s+1,t+1}}{(1+r^e_t)(1+r^e_{t+1})} - \ldots - \frac{O^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t}(1+r^e_j)} \end{aligned}$$

When the asset is one year older, the services it renders in year t will have been received and the operating expenses of year t already incurred. Consequently, the expected price of the asset at the beginning of year t+1 is given by

(A2)
$$P^e_{s+1,t+1} = \frac{C^e_{s+1,t+1}}{1+r^e_{t+1}} + \frac{C^e_{s+2,t+2}}{(1+r^e_{t+1})(1+r^e_{t+2})} + \ldots + \frac{C^e_{m-1,t+m-s-1}}{\Pi_{j=t+1}^{t+m-s-1}(1+r^e_j)}$$
$$- \frac{O^e_{s+1,t+1}}{1+r^e_{t+1}} - \frac{O^e_{s+2,t+2}}{(1+r^e_{t+1})(1+r^e_{t+2})} - \ldots - \frac{O^e_{m-1,t+m-s-1}}{\Pi_{j=t+1}^{t+m-s-1}(1+r^e_j)}$$

Dividing both sides of equation (A2) by $(1+r^e_t)$ and subtracting the result from equation (A1) yields

(A3)
$$P_{s,t} - \frac{P^e_{s+1,t+1}}{1+r^e_t} = \frac{C^e_{s,t}}{1+r^e_t} - \frac{O^e_{s,t}}{1+r^e_t}$$

Multiplying both sides of equation (A3) by $(1+r^e_t)$ and combining terms, one obtains the standard user cost measure:

(A4)
$$C^e_{s,t} = r^e_t P_{s,t} + (P_{s,t} - P^e_{s+1,t+1}) + O^e_{s,t}$$

Equation (A4) expresses the expected value of the asset's services as the sum of three components: the expected nominal net operating surplus, the expected decline in the price of the asset during the year, and the expected value of operating expenses. The expected decline in the price of the asset is usually partitioned into two components: depreciation and (the negative of) the expected capital gain on the asset.

The astute reader will recognize that while I assume that the purchaser has expectations regarding all future discount rates and prices that will be in effect during the asset's service life, the derivations of equations (A3) and A(4) demonstrate that the relevant parts of all of that information is contained in information pertaining to year t. Specifically, it is contained in the purchaser's expectations of the level of operating expenses (by vintage) that will be in effect during year t and the levels of prices (by vintage) that will be in effect at the end of that year (and, therefore, the beginning of year t+1).

Appendix B – Obtaining Implied Age-price Profiles From Age-efficiency Profiles

An asset's age-efficiency profile measures the value of the services of an asset at various ages relative to the value of the services it produced when new, all services being measured in terms of rental prices of the reference year. The relative efficiency profile $\phi(s)$ is, thus, a schedule of m values, one for each possible age of the asset, that gives the ratio of the (expected) net service value of an s-year old asset to the service value that the asset would have had if it were new, i.e., 0-years old. Thus, we have

(B1) $\phi(s) = C^e_{s,b} \,/\, C^e_{0,b}$ where b is the reference year.

This ratio is unaffected by changes in the rental price of a new asset (of the given type) over time so that we have

(B2) $C^e_{s,b} \,/\, C^e_{0,b} \;=\; C^e_{s,t} \,/\, C^e_{0,t} \;=\; \phi(s)$

To show how the age-price profile can be derived from the associated relative efficiency profile, let us note that, by definition, the expected rate of inflation during year t in the price of the given type of asset, $\dot{P}^e_{0,t}$, is given by

(B3) $(1+\dot{P}^e_{0,t}) \;\equiv\; P^e_{0,t+1} / P_{0,t}$

Nominal values of the services of a new asset are expected to inflate at the same rate as the price of a new asset. Therefore, we have

(B4) $C^e_{0,\,j+1} = (1+\dot{P}^e_{0,\,j})\, C^e_{0,\,j}$

If we assume that there are no operating expenses associated with the asset and then substitute equation (B2) into equation (A1), we obtain

(B5) $$P_{s,t} = \frac{\phi(\mathrm{s})\;\; C^e_{0,t}}{1+r^e_t} + \frac{\phi(\mathrm{s}+1)\, C^e_{0,t+1}}{(1+r^e_t)\,(1+r^e_{t+1})} + \ldots + \frac{\phi(\mathrm{m}-1)\, C^e_{0,t+m-s-1}}{(1+r^e_t)\,\Pi^{t+m-s-1}_{i=t+1}(1+r^e_i)}$$

By repeatedly substituting equation (B4) into equation (B5) we obtain

(B6) $$P_{s,t} = \frac{\phi(\mathrm{s})\;\; C^e_{0,t}}{1+r^e_t} + \frac{\phi(\mathrm{s}+1)\, C^e_{0,t}(1+\dot{P}^{\,e}_{0,\,t+1})}{(1+r^e_t)\,(1+r^e_{t+1})} + \ldots + \frac{\phi(\mathrm{m}-1)\, C^e_{0,t}\,\Pi^{t+m-s-1}_{i=t+1}(1+\dot{P}^{\,e}_{0,\,i})}{(1+r^e_t)\,\Pi^{t+m-s-1}_{i=t+1}(1+r^e_i)}$$

We define the expected real own rate of interest for assets of the given type during year t, ρ^e_t, by

(B7) $$(1 + \rho_t^e) \equiv (1 + r_t^e)/(1 + \dot{P}_{0,t}^e)$$

Substituting this into equation (B6) we obtain

(B8) $$P_{s,t} = \frac{\phi(\mathrm{s})\ C_{0,t}^e}{1+r_t^e} + \frac{\phi(\mathrm{s}+1)\, C_{0,t}^e}{(1+r_t^e)\,(1+\rho_{t+1}^e)} + \ldots + \frac{\phi(\mathrm{m}-1)\, C_{0,t}^e}{(1+r_t^e)\,\Pi_{i=t+1}^{t+m-s-1}(1+\rho_i^e)}$$

Equation (B8) is really a system of m equations, one for each possible age of the asset. To obtain the age-price profile we take the right hand side of (B8) for any given age of the asset and divide it by the comparable expression for new assets. The value of $C_{0,t}^e$ in the numerator and denominator of the resulting quotient cancel out and the values of all of the other variables are known by assumption. Consequently, we are able to compute the value of the quotient, which gives us one point on the age-price profile. We then repeat this procedure for all other possible ages of the asset.

Appendix C - Proof of Proposition that a Change in the Real Own Rate of Interest Will Cause Either the Age-Efficiency or Age-Price Profile to Change.

The standard user cost expression for the expected services of a durable that is s-years old in year t is given by $C_{s,t}^e = r_t^e P_{s,t} + (P_{s,t} - P_{s+1,t+1}^e)$, where $P_{s,t}$ denotes the purchase price of an s-year old asset at the beginning of year t; $P_{s+1,t+1}^e$ denotes its expected purchase price at the beginning of year t+1 when the asset is one year older; $C_{s,t}^e$ denotes the expected value of the services of this s-year old asset in year t; and r_t^e denotes the expected nominal discount rate in year t.

Let us assume that the asset in question is new in year t and has a service life of 2 years. (We can do this without loss of generality as long as the service life is finite. When depreciation is strictly geometric, service lives are infinite so that this proof does not apply to such assets. In fact the proposition does not hold them.) The service value of this asset in year t is given by $C_{0,t}^e = r_t^e P_{0,t} + (P_{0,t} - P_{1,t+1}^e)$. The service value of a 1-year old version of this asset in year t is given by $C_{1,t}^e = r_t^e P_{1,t} + P_{1,t}$ because this asset has a value of 0 at the end of year t when it is two years old. The relative efficiency of the 1-year old asset in year t is given by $\phi(1) = C_{1,t}^e / C_{0,t}^e$, the ratio of the two service values. If the age-efficiency profile of the asset is a constant over time and unaffected by changes in interest rates, then ϕ(1) will be a constant, which we denote by *a*. Thus, we have

(C1) $$a = C^e_{1,t} \;/\; C^e_{0,t} = (r^e_t P_{1,t} + P_{1,t}) \;/\; (r^e_t P_{0,t} + (P_{0,t} - P^e_{1,t+1}))$$

Now, if the asset's age-price profile is also constant over time, then $P_{1,t} = bP_{0,t}$ where *b* is a constant over time. We also assume that this relationship holds for expected as well as actual prices. Substituting this into equation (C1) and multiplying both sides of the equation by the denominator on the right hand term, we have

(C2) $$a\,(r^e_t P_{0,t} + P_{0,t} - bP^e_{0,t+1}) = (1+r^e_t)\,b\;P_{o,t}$$

Let the expected rate of inflation in the price of the asset in year t be denoted by ρ^e_t so that we have $P^e_{0,t+1} = (1+\rho^e_t)\,P_{0,t}$. Substituting this into (C2), we obtain

(C3) $$a\,(r^e_t P_{0,t} + P_{0,t} - b(1+\rho^e_t)\;P_{0,t}) = (1+r^e_t)\,b\;P_{o,t}$$

Dividing through both sides of (C3) by $P_{0,t}$, we have

(C4) $$a\,((1+r^e_t) - b(1+\rho^e_t)) = b\;(1+r^e_t)$$

By moving (by addition) the rightmost part of the expression in parenthesis of the left hand side of (C4) to the right hand side of the equation and by moving what had been the right hand side of the equation to the left hand side, we obtain

(C5) $$(a - b)\;(1+r^e_t) = a\,b(1+\rho^e_t)$$

We then divide both sides of equation (C5) by *a b* $(1+r^e_t)$ to obtain

(C6) $$\frac{a-b}{ab} = \frac{1+\rho^e_t}{1+r^e_t}$$

Because *a* is a constant and *b* is a constant, the product of *a* and *b* is a constant and the difference between a and b is also a constant. This means that the expression on the right hand side of equation (C6) must also be a constant. But, that expression is the real own rate of interest. If this real own rate is not a constant, then one of our assumptions about *a* and *b* both being constant must be wrong. This demonstrates that either the age-efficiency schedule or the age-price schedule must change if the real own rate of interest changes.

Note that this proof holds for all assets that have finite service lives. All such assets will eventually reach the point where they have two years left in their services lives and the actual conditions used in this proof will hold.

References

Böhm-Bawerk, E. V. (1891), *The Positive Theory of Capital*, W. Smart (translator of the original German book published in 1888), New York: G.E. Stechert.

Faucett, Jack G. 1980. "Comment," In *The Measurement of Capital*, Studies in Income and Wealth, vol. 45, ed., Dan Usher, 68-81. Chicago: University of Chicago Press for the National Bureau of Economic Research.

Fraumeni, Barbara M. 1997. "The Measurement of Depreciation in the U.S. National Income and Product Accounts." *Survey of Current Business, 77(7)*: 7-23.

Harper, Michael J. 1982. "The Measurement of Productive Capital Stock, Capital Wealth, and Capital Services." BLS Working Paper 128, June.

Harper, Michael J., Brent R. Moulton, Steven Rosenthal and David B. Wasshausen. 2009. "Integrated GDP-Productivity Accounts," *The American Economic Review*, 99(2): 74-79.

Hicks, John R. 1965. Value and Capital, 2nd ed. (First published in 1946.) London: Oxford University Press.

Hulten, Charles R., and Frank C. Wykoff. 1980. "Economic Depreciation and the Taxation of Structures in United States Manufacturing Industries: An Empirical Analysis," In *The Measurement of Capital*, Studies in Income and Wealth, vol. 45, ed., Dan Usher, 83-109. Chicago: University of Chicago Press for the National Bureau of Economic Research.

Hulten, Charles R., and Frank C. Wykoff. 1981. "The Measurement of Economic Depreciation." In *Depreciation, Inflation, and the Taxation of Income from Capital*, ed., Charles R. Hulten, 81–125. Washington, DC: The Urban Institute Press.

Jack Faucett Associates. 1967. "Methodology for Constructing Gross and Net Capital Stock Series for Input-Output Sectors," Report prepared for the Bureau of Labor Statistics, September

Jorgenson, Dale W. 1989. "Capital as a Factor of Production." In *Technology and Capital Formation,* eds., Dale W. Jorgenson and Ralph Landau, 1-35. Cambridge and London: the MIT Press.

Katz, Arnold J. 1982. "The Value of Services Provided by the Stock of Consumer Durables, 1947-79: Alternative User Cost Measures." In *BEA Working Paper No. 2: Measuring Nonmarket Economic Activity,* Accession No. PB 83-167-395. Springfield, Va.: National Technical Information Service, December.

Katz, Arnold J. 2003. "*Estimating Dwelling Services in the Candidate Countries: Theory and Application of the User Cost of Capital Measure.*" Paper presented at the Regional Coordinators' Workshop of the International Comparison Program held in Luxembourg, March 24 – 28.

Katz, Arnold J. 2008. "Accounting for Obsolescence: An Evaluation of Current NIPA Practice." Paper presented at the 2008 World Congress on National Accounts for Nations held in Arlington, VA, May 12-17. (Published as BEA Working Paper WP-2008-05).

Katz, Arnold J. 2009. "Estimating Dwelling Services in the Candidate Countries: Theoretical and Practical Considerations in Developing Methodologies Based on a User Cost of Capital Measure," chapter 3, pp. 33-50 in W. Erwin Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox, and Alice O. Nakamura, eds. (2009). *Prices and Productivity Measurement: Volume 1 – Housing.* Trafford Press. Also available as a free e-publication at www.vancouvervolumes.com and www.indexmeasures.com.

Katz, Arnold J. and Shelby W. Herman. 1997. "The Measurement of Depreciation in the U.S. National Income and Product Accounts." *Survey of Current Business, 77(5)*: 69-89.

Sliker, Brian K. 2012. "How to Calculate a Declining Balance Rate from Service-Life Data." Unpublished paper, February 14.

OECD Manual. 2009. *Measuring Capital, 2nd edition.*

U.S. Department of Commerce, Bureau of Economic Analysis. 2003. *Fixed Assets and Consumer Durable Goods in the United States, 1925-99*. Washington, DC: USGPO.

U.S. Department of Commerce, Bureau of Economic Analysis. 2011. *NIPA Handbook: Concepts and Methods of the U.S. National Income and Product Accounts*. Available online at http://www.bea.gov/methodologies/index.htm#national_meth

U.S. Department of Labor, Bureau of Labor Statistics. 1970. *Capital Stock, Production Functions, and Investment Functions for Selected Input-Output Sectors*, BLS Bulletin 355. Washington, D.C.: USGPO, February.

U.S. Department of Labor, Bureau of Labor Statistics. 1979. *Capital Stock Estimates for Input-Output Industries: Methods and Data*, BLS Bulletin 2034. Washington, D.C.: USGPO, September.

U.S. Department of Labor, Bureau of Labor Statistics. 1983. *Trends in Multifactor Productivity, 1948-81*, BLS Bulletin 2178, Washington, D.C.: USGPO, September.

Young, Allan H. and John C. Musgrave. 1980. “Estimation of Capital Stock in the United States,” In *The Measurement of Capital*, Studies in Income and Wealth, vol. 45, ed., Dan Usher, 23-82. Chicago: University of Chicago Press for the National Bureau of Economic Research.