Methods of Temporal Disaggregation for Estimating Output of the Insurance Industry

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Abstract

A variety of mathematical and statistical methods have been developed and applied by researchers to solve problems of temporal disaggregation, the process of estimating unobserved sub-annual series from observed annual values. Despite a vast body of work evaluating the ability of different mathematical and statistical methods to accurately estimate the temporal dynamics of target series, few empirical papers have attempted to establish the conditions under which some of these methods may have an advantage over competing models. While most empirical studies have focused on applying these methods to relatively well-behaved series, this paper examines to what extent the volatility of the target series being estimated may be a factor in the relative performance of four different mathematical methods of temporal disaggregation: the Denton proportional first difference method (with and without a related indicator series), the Causey-Trager growth rate preservation method, and the cubic spline interpolation method. Using source data provided by the National Association of Insurance Commissioners (NAIC) for twenty-three lines of property and casualty insurance between 2002 and 2012, we employ each of the four methods to estimate quarterly output by line of insurance for BEA's Industry Economic Accounts. To our knowledge, this is the first empirical study of its kind to use insurance industry data to examine the performance of these methods as it relates specifically to the volatility of the target series.

1 Introduction

The Industry Economic Accounts directorate of the U.S. Bureau of Economic Analysis (BEA) has recently begun publishing quarterly statistics of gross output and value added by industry. In some industries, a lack of timely and reliable source data can pose a challenge to economists producing estimates of this nature, as it is common that only annual source data are available at the time the estimates are produced. One commonly used mathematical tool for addressing this challenge in time series work is temporal disaggregation, the process of estimating unobserved sub-annual series from observed annual values.

A variety of mathematical and statistical methods have been developed and applied by researchers to solve problems of benchmarking and temporal disaggregation. There is a vast body of work evaluating the ability of the different methodologies to accurately estimate the temporal dynamics of target series, both on the basis of accuracy and computational efficiency.[1] Chen (2007) and Brown (2012) each compare the performance of three mathematical methods of temporal disaggregation with related indicator series, specifically, the Denton PFD method, Causey-Trager growth rate preservation method, and the cubic spline interpolation method.[2] They both conclude that the Denton PFD and Causey-Trager growth rate preservation models are most accurate at estimating sub-annual series, but they are split on which of the two is the preferred method, with Chen finding the Denton PFD method to perform best and Brown favoring the Causey-Trager model (Brown, 2012; Chen 2007).

Other studies have examined the performance of temporal disaggregation methods in the absence of a related indicator series, typically setting the indicator series to a constant value. Abeysinghe and Lee (1998) compared the Chow-Lin (1971) regression method, which requires a related series, to a univariate spline interpolation which does not.[3] In disaggregating Malaysian annual GDP into six sectors on a quarterly basis, they found that using a related series produced the more favorable results in their final estimated series (Abeysinghe and Lee, 1998). By contrast, Quenneville, Picard, and Fortier (2013), using the Industrial Production Index from the Federal Reserve Board as their sub-annual indicator series for disaggregating annual U.S. GDP into quarters, found that the spline method was not only computationally more efficient, but that use of a constant indicator was preferable to the use of the related series across the four methods they evaluated.[4] They were careful to note, however, that use of the indicator series should be evaluated for individual needs, as it may be reasonable for a constant indicator to have a superior performance simply due to a changing relationship between the target series and the related series (Quenneville et al., 2013).

Despite the number of empirical studies conducted to evaluate the merits of various methods of temporal disaggregation, there is no consensus that one method is consistently superior in all situations. Rather, a common conclusion is that the choice of method depends on the desired application. However, few empirical papers have attempted to establish the conditions under which some of these methods may have an advantage over competing models. Most empirical studies have focused on applying these methods to relatively well-behaved series; for example, constructing quarterly estimates of GDP (Abeysinghe and Lee, 1998; Di Fonzo and Marini, 2005a; Trabelsi and Hedhili, 2005) manufacturing (Brown, 2012), or retail and wholesale trade data (Brown, 2012; Dagum and Cholette, 2006; Di Fonzo and Marini, 2005b) from observed annual levels.

Using source data for twenty-three lines of business for the property and casualty (P&C) insurance industry between 2002 and 2012 provided by the National Association of Insurance Commissioners (NAIC), this paper examines to what extent the volatility of the target series being estimated may be a factor in the relative performance of four different mathematical methods of temporal disaggregation: the Denton proportional first difference method (with and without a related indicator series), the Causey-Trager growth rate preservation method, and the cubic spline interpolation method. The dataset consists of both large, well-behaved series and relatively small, more volatile series, all of which must be solved as a system subject to both temporal (one series observed over time) and contemporaneous (many series observed at one point in time) constraints. We assess the ability of each method to mimic the temporal dynamics of the target series in terms of level accuracy, growth rate preservation, and the ability to predict turning points in the series, and rank the performance of each method within volatility quintiles. This information is used to determine the most suitable method(s) for estimating unobserved

[1] For an overview of the existing research conducted on interpolation, temporal disaggregation, and extrapolation, see Pavía-Miralles (2010).

[2] Chen (2007) also includes five statistical methods, including extensions of the Chow-Lin regression method. For a complete description of the five statistical methods and their relative performances, see Chen (2007).

[3] Bloem, Dipplesman, and Maehle (2001) note that the Chow-Lin regression method is not a true benchmarking method, although it does make use of at least one related indicator series.

[4] Their study compares the cubic spline method (with and without a related series), the Denton PFD method, and a state space model. State space models provide the basis for Denton's benchmarking method (Quenneville et al., 2013). See Durbin and Koopman (2012) for a complete discussion on the use of state space models in time series analysis.

components of insurance industry output for the purposes of BEA's quarterly GDP-by-industry statistics. To our knowledge, this is the first empirical study of its kind to use insurance industry data to examine the performance of these methods as it relates specifically to the volatility of the target series.

The remainder of this paper is structured as follows: Section 2 details different mathematical methods of temporal disaggregation. Section 3 describes our proposed two-stage Denton PFD method, as it is applied to our dataset. Section 4 presents our empirical results and Section 5 contains an analytical discussion and concluding remarks.

2 Three Mathematical Methods of Temporal Disaggregation

2.1 The Modified Denton Benchmarking Procedure

One of the most widely-used mathematical methods for disaggregating and benchmarking time series was first proposed by Frank Denton in 1971, and subsequently modified by Helfand, Nash, and Trager (1977) and Cholette (1984). Denton (1971) presented a quadratic minimization approach for benchmarking a sub-annual series to annual totals, with the goal of preserving the original movements between sub-annual periods. His approach is a generalization of the method presented by Boot, Feibes, and Lisman (1967) for creating a sub-annual series from annual levels, when no preliminary sub-annual indicator series exists (Denton, 1971). Of Denton's movement preservation methods, the proportional first difference variant is most commonly applied for purposes of benchmarking and temporal disaggregation.[5] This method seeks to choose a set of sub-annual estimates that satisfy a set of annual benchmark constraints while minimizing the relative difference between the estimated series and the preliminary series (Chen, 2007; Dagum and Cholette, 2006; Denton, 1971).

While Denton's method was intended to support the goal of movement preservation, his choice of initial conditions was later shown by Cholette (1984) to introduce transient movements at the beginning of the benchmarked series, thereby violating the very principle it sought to maintain (Brown, 2012; Chen and Andrews, 2008; Dagum and Cholette, 2006). In order to reduce the dimension of the problem, Denton (1971) imposed an initial condition that the first observation of the benchmarked series be equal to the first observation of the preliminary series: $x_0 = p_0$. Cholette (1984) corrected for these transient movements by removing Denton's initial condition and restoring the first differences matrix to its original form.[6]

Seeking to estimate a series of n sub-annual values given a set of N annual observations, Denton's (1971) PFD method chooses the (n x 1) vector of sub-annual estimates ($\mathbf{x}$) that minimizes the objective function specified in (1), subject to an appropriately chosen aggregation constraint:

$$\min_{x_t} f_{\text{PFD}}(x, p) = \sum_{t=2}^{n} \left(\frac{x_t}{p_t} - \frac{x_{t-1}}{p_{t-1}}\right)^2 \; s.t. \; \mathbf{Ax} = \mathbf{b} \tag{1}$$

That is, letting $\mathbf{x}$ and $\mathbf{p}$ be defined as vectors of the benchmarked sub-annual estimates and the preliminary (indicator) series, respectively, the Denton PFD method seeks to find target values that make the proportional period-to-period differences between the benchmarked and preliminary series as small as possible. The aggregation constraint is defined by a vector of annual observations, ($\mathbf{b}$), treated as binding constraints, and a (N x n) temporal aggregation matrix, expressed as $\mathbf{A} = \mathbf{I}_N \otimes \mathbf{a}'$, with $\mathbf{a} = (\frac{1}{s})1_s$, where s is defined as the number of sub-annual periods in each year.[7] This can be rewritten in matrix notation as:

$$\min_{\mathbf{x}} f_{\text{PFD}}(\mathbf{x},\ \mathbf{p}) = (\mathbf{x} - \mathbf{p})'\mathbf{Q}(\mathbf{x} - \mathbf{p}) \; s.t. \; \mathbf{Ax} = \mathbf{b}, \tag{2}$$

which can be expressed compactly, as the solution to the following linear system:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{A}' \\ \mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix} \tag{3}$$

[5] Chen (2007) provides details on three other variants of Denton's method: the additive first difference, additive second difference, and proportional second difference. However, these are used to a much lesser extent in the literature.

[6] Due to its ease of use, our notation follows closely that of Daalmans and Di Fonzo (2014).

[7] This notation assumes stock data, where sub-annual estimates should average to the annual constraint. For flow data, where sub-annual estimates should sum to the annual constraint, this would simplify to a (s x 1) vector of ones.

where $\mathbf{Q} = \hat{\mathbf{P}}^{-1}\Delta_n'\Delta_n\hat{\mathbf{P}}^{-1}$, $\hat{\mathbf{P}} \equiv diag(\mathbf{p})$, λ is a (N x 1) vector of Lagrange multipliers, and $\mathbf{0}$ is defined as a matrix of zeros of appropriate dimension.

Finally, Δ_n is the modified (($n-1$) x n) first differences matrix, as proposed by Cholette (1984):

$$\Delta_n = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

Solving (3) yields a (($n+N$) x 1) vector containing the desired n sub-annual estimates:

$$\begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q} & \mathbf{A}' \\ \mathbf{A} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix} \tag{4}$$

2.2 The Causey-Trager Growth Rate Preservation Model

The Causey-Trager (1981) growth rate preservation model (GRP) is a mathematical method for benchmarking and temporal disaggregation; it is used by the U.S. Census Bureau when benchmarking sub-annual estimates to annual survey data and when benchmarking annual time series to the Economic Census every five years (Brown 2012).[8]

Seeking to estimate a series of n quarterly values given a set of N annual observations, the GRP method chooses the (n x 1) vector of sub-annual estimates, ($\mathbf{x}$), that solves the non-linear minimization problem specified in (5), where all variables are defined as in Section 2.1:

$$\min_{x_t} f_{\text{GRP}}(x,p) = \sum_{t=2}^{n} \left(\frac{x_t}{x_{t-1}} - \frac{p_t}{p_{t-1}}\right)^2 \; s.t. \; \mathbf{Ax} = \mathbf{b} \tag{5}$$

Since the objective function explicitly seeks to minimize revisions to the period-to-period growth rates of the indicator series, the Causey-Trager GRP method is sometimes referred to in the literature as the "ideal" benchmarking method (Di Fonzo and Marini, 2013; Temurshoev, 2012). However, this adherence to the temporal dynamics of the indicator series comes at the cost of computational efficiency. Unlike the Denton PFD optimization problem in Section 2.1, which has linear first-order conditions, and hence a convenient closed-form solution as shown in (4), the minimization problem specified in (5) is inherently non-linear and requires the use of numerical methods to find a solution (Bozik and Otto, 1988; Brown, 2012; Daalmans and Di Fonzo, 2014; Di Fonzo and Marini, 2013; Temurshoev, 2012). Causey and Trager (1981) employed a non-linear programming method using a steepest feasible descent algorithm in order to find a local minimum for the optimization problem posed in (5); they proposed using the solution to the Denton PFD method obtained in (4) as the initial condition because it can be viewed as a close approximation to the growth rate of the indicator series (Brown, 2012; Dagum and Cholette, 2006; Di Fonzo and Marini, 2013).

As a result of the non-linearity of the problem and the subsequent lack of a closed-form solution, the GRP method is more computationally intensive to execute than the Denton PFD method. When applying the GRP method, researchers can choose from a variety of non-linear optimization algorithms, and in most cases seem to utilize the stock algorithms available in their software package of choice. Di Fonzo and Marini (2013) and Temurshoev (2012) both utilized MatLab's stock *fmincon* function (Mathworks, 2014), choosing to employ its *interior-point* and *Sequential Quadratic Programming (SQP)* algorithms, respectively. In practice, a researcher's choice of a non-linear solver is more a matter of personal preference and is likely to be of little consequence for the purpose of the applications discussed in this paper.

[8] The work of Causey and Trager (1981), namely the development of the numerical algorithm employed at the U.S. Census Bureau for solving this non-linear constrained optimization problem, actually appears as unpublished research notes in an appendix to a research report by Bozik and Otto (1988) (Brown, 2012).

2.3 Spline Interpolation Methods

Another mathematical approach to temporal disaggregation uses a cubic spline interpolation method to derive sub-annual estimates from annual observations. The cubic spline interpolation is a special case of a univariate smoothing method proposed and applied to the problem of temporal disaggregation by Boot et al. (1967) (Quenneville et al., 2013). The cubic spline method can be implemented with or without the use of a sub-annual indicator series (Chen, 2007; Quenneville et al. 2013) and is easily executed using stock procedures in most commercially-available mathematical and statistical packages. In the absence of an indicator series, Chen (2007) states that "the unknown sub-annual trend can be conveniently described by a mathematical function of time." These qualities make the cubic spline a potentially attractive option in applications where adequate indicator series are not available and computational efficiency is the primary concern.

In the context of temporal disaggregation, the goal of the cubic spline method is to fit a series of third-degree polynomial functions between annual observations and join them in such a way that the resulting function taken as a whole (the spline function) has continuous first and second derivatives (Brown, 2012). Quenneville et al. (2013) showed that the spline interpolation is a special case of Denton's (1971) method. When the indicator series is set equal to a vector of ones (that is, when no related indicator series is available), the solution to the spline interpolation is the continuous limit of the additive first difference variant;[9] when a related indicator series is used, the solution to the spline interpolation is the continuous limit of a weighted form of the PFD variant with the weights equal to the inverse of the indicator series (Quenneville et al., 2013).

3 Deriving Quarterly Estimates of Insurance Premiums

Researchers have evaluated the relative merits of each of the methods discussed in Section 2 in a variety of circumstances, using both real and simulated data. The empirical literature on temporal disaggregation of economic time series has thus far primarily focused on applying these techniques to well-behaved, aggregate series. For example, many researchers have applied these techniques to the quarterly disaggregation of annual GDP, which for most developed countries does not tend to be highly volatile (Abeysinghe and Lee, 1998; Di Fonzo and Marini, 2005a; Trabelsi and Hedhili, 2005). Brown (2012) applies the methods discussed here to manufacturing, retail, and wholesale trade data from the U.S. Census Bureau. Similarly, Dagum and Cholette (2006) and Di Fonzo and Marini (2005b) apply these methods to systems of Canadian retail trade series.[10] While a common conclusion in these comparative studies is that the choice of method is situation-dependent, few papers have attempted to establish the conditions under which some of these methods may have an advantage over competing models. In the remainder of this paper, we seek to understand the extent to which the volatility of the target series being estimated may be a factor in the relative performance of the methods discussed in Section 2 by examining how each of the methods performs in the context of time series exhibiting varying degrees of volatility.

We examine property and casualty (P&C) insurance data provided by the National Association of Insurance Commissioners (NAIC), which is used by the U.S. Bureau of Economic Analysis (BEA) in the estimation of quarterly output for the insurance industry. BEA's methodologies for estimating quarterly and annual output for the P&C industry, which are presented in Appendix A, require timely and reliable estimates of premiums, losses, and investment gains. The NAIC data contains premium levels for twenty-three sub-sectors, or lines of business (LOB), on both a quarterly and annual basis for the years 2002 through 2012. However, while data for losses and investment gains are available at the aggregate sector level on both a quarterly and annual basis, the dataset does not contain quarterly observations of these output components by LOB. Thus, estimating quarterly P&C output by LOB requires a method for deriving more detailed time series for losses and investment gains at the desired quarterly frequency.

We apply the temporal disaggregation methods discussed in Section 2 in order to construct quarterly premium estimates for each sub-sector using only the level of detail available to us for losses and investment gains. We use annual premiums by LOB and aggregate premiums for the P&C sector to specify temporal and contemporaneous benchmarking constraints, respectively. We compare our predicted quarterly levels for each LOB to the observed

[9] The spline method is also the continuous limit of the Boot et al. (1967) method when the indicator series is given as a vector of ones (Quenneville et al., 2013).

[10] In a similar study, Di Fonzo and Marini (2005a,b) disaggregated a system of time series, using annual Industrial Value Added figures from Italy into quarterly estimates for six sectors. The authors also implemented Denton PFD method, this time comparing it with a data-based signal extraction approach proposed by Guerrero and Nieto (1999), which also uses related indicator series. Their research concluded there was no "perceivable" difference in the levels of the final benchmarked series produced by the Denton PFD and Guerrero-Nieto methods (Di Fonzo and Marini, 2005a,b).

premium levels provided by NAIC, and construct various statistics to quantify the accuracy of the different methods; we use these results to identify the most appropriate method(s) for estimating quarterly insurance losses, investment gains, and therefore output, in the Industry Economic Accounts.

Due to the relatively small and inconsistent nature of some sub-sectors of the P&C industry, our dataset contains a mix of both large, well-behaved series and smaller, more volatile series. As such, this dataset is useful for evaluating the relative performance of each of the aforementioned temporal disaggregation methods when confronted with a system of time series of this nature, where both temporal and contemporaneous constraints must hold.

The next section provides details on the two-stage Denton PFD method that we use to estimate quarterly premiums by LOB. We compare these estimates with those generated by the Causey-Trager GRP and cubic spline methods discussed in Sections 2.2 and 2.3. Consistent with Di Fonzo and Marini (2013), we choose to implement the GRP method by utilizing the *Sequential Quadratic Programing (SQP)* algorithm available in the *fmincon* function of MatLab's *Optimization Toolbox* (Mathworks, 2014). We used default settings in our procedure, with the exceptions that the maximum number of iterations were set at $5,000$, and the tolerance levels for the choice variable, the function, and the constraint were all set to $1e^{-10}$.[11] Several studies including Abeysinghe and Lee (1998), Brown (2012), and Quenneville (2013) employ univariate cubic spline interpolation methods as a means for conducting temporal disaggregation. For the estimates derived using this technique, we utilize the stock SAS procedure, *proc expand*, with default settings (SAS Institute, Inc., 2011a), to disaggregate annual observations into quarterly observations.

3.1 Our Proposed Two-Stage Denton PFD Method

Our methodology for estimating quarterly premiums by LOB from annual levels represents an adaptation of a method proposed in Abeysinghe and Lee (1998), who use a univariate (spline) method of temporal disaggregation to derive estimates of quarterly GDP by industry for Malaysia from annual sectoral shares.[12] In order to construct a quarterly series of insurance premiums using only annual levels and quarterly aggregates, we employ a two-stage, modified Denton PFD procedure, under two different sets of assumptions. As a preliminary step, we use the *proc X12* procedure in SAS, an implementation of the U.S. Census Bureau's X12 ARIMA software (SAS Institute, Inc. 2011b; U.S. Census Bureau, 2011), to seasonally adjust both the aggregate quarterly premiums for the P&C industry (to be used as an indicator series) as well as the true quarterly LOB premiums (against which we will test our quarterly estimates).[13] We also calculate annual premium shares for each LOB from annual NAIC data.

In the first stage, we apply the modified Denton PFD method presented in Section 2.1 in order to estimate an initial set of quarterly premium shares by LOB. We specify the objective function as in equations (1) and (2), letting $\mathbf{x}$ and $\mathbf{p}$ be defined as (44×1) vectors of estimated premium shares and indicator values, respectively. For our first set of estimates, denoted D_{rel} to indicate the use of a related series in the Denton procedure, we assume that the data generating process underlying premiums for each line of business is the same as that underlying the aggregate quarterly series. Under this assumption we use the aggregate premium levels as the (related) indicator series for estimating premium shares for each LOB. For our second set of estimates, denoted D_{cons}, we use a constant vector of ones as the indicator series for estimating premium shares for each LOB. Implicitly, this assumes that we have no information about the quarterly movements of the true series, which allows the information contained only in the annual observations to dictate the movements of the estimated series. The objective function in this case reduces to minimizing the sum of squared differences in quarterly shares, subject to binding constraints in each year. With eleven years of quarterly data ($N = 11$ and $n = 44$), it follows that the first differences matrix, Δ_n, is a (43×44) matrix.

Under both sets of assumptions, the aggregation constraint is the same; for each LOB we force the estimated quarterly shares to average to the known premium share, annually. Thus, applying the constraint from equation (1) we have, $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a matrix of dimension (11×44), defined as before, and $\mathbf{a}$ is now defined as a

[11] Tolerance levels out to $1e^{-15}$ were tested, but resulted in indistinguishable differences to the estimated shares.

[12] Abeysinghe and Lee (1998) calculate annual shares of GDP for eight sectors of the Malaysian economy, and use a stock univariate (spline) interpolation procedure available in SAS to derive estimates of quarterly sectoral shares. Those shares are then applied to seasonally-adjusted, quarterly estimates of aggregate GDP for Malaysia in order to construct estimates of seasonally-adjusted, quarterly GDP by industry. The authors note that this shares-based approach can only be applied to seasonally adjusted quarterly aggregates; multiplying the estimated quarterly shares by the seasonally *unadjusted* quarterly GDP series would force the seasonal pattern of each estimated series to be the same as that of the aggregate series. Note also that our method is in some ways similar to Trabelsi and Hedhili (2005) except that they use a regression approach.

[13] Following Abeysinghe and Lee, 1998; Di Fonzo and Marini, 2005a; Trabelsi and Hedhili, 2005, we seasonally adjust our data before performing any estimation.

(4×1) vector with each element equal to $\frac{1}{4}$.

As suggested by Chen (2007), we then use the preliminary shares estimated in the first stage and employ a second-stage modified Denton PFD procedure, this time specified to satisfy the contemporaneous constraint that the sum of premium shares across all lines in a given quarter sum to one. Mathematically, if we let $i \in \{1, 2, ..., k\}$ index each LOB, then for each quarter $t \in \{1, 2, ..., n\}$ the second stage can be expressed as:

$$\min_{y_i} f_{\text{PFD}}(y, x) = \sum_{i=2}^{k} \left(\frac{y_{i,t}}{x_{i,t}} - \frac{y_{i-1,t}}{x_{i-1,t}}\right)^2 \; s.t. \; \sum_{i=1}^{k} y_{i,t} = 1 \tag{6}$$

where $\mathbf{y}_t$ is a vector of quarterly premium shares, adjusted to satisfy the contemporaneous constraint in quarter t, and $\mathbf{x}_t$ is the vector of initial shares estimated in the first stage. From the steps outlined in Section 2.1, the second stage objective has a familiar closed-form solution, found by solving the following linear system:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{A}' \\ \mathbf{A} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{y}^{PFD} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{7}$$

where the notation is similar to that in Section 2.1, except that $\mathbf{Q}$ is now the $(k \times k)$ matrix defined as $\mathbf{Q} = \hat{\mathbf{X}}^{-1} \Delta_k' \Delta_k \hat{\mathbf{X}}^{-1}$, $\mathbf{A}$ is now a $(1 \times k)$ vector of ones, and $\mathbf{0}$ is a $(k \times 1)$ vector of zeros. The first differences matrix in this stage reduces in dimension to $((k-1) \times k)$.

The second stage process is run iteratively for each quarter, with the adjusted premium shares then transposed and written to a final output file. Estimates of quarterly, seasonally-adjusted premium levels are then calculated by multiplying the estimated quarterly shares for each LOB by the seasonally-adjusted aggregate premiums for the P&C industry.

One advantage of the two-stage Denton PFD method, is its built-in ability to distribute contemporaneous discrepancies, avoiding the need to distribute them in a subjective, manual manner (Chen and Andrews, 2008).[14] However, it should be noted that any procedure used to satisfy these constraints will result in slight deviations from the temporal constraints imposed in the first stage. This can be reconciled iteratively by running successive Denton procedures until a given tolerance is achieved or by employing an iterative proportional fitting approach, such as that recommended by Dagum and Cholette (2006). While this may be deemed necessary in some applications, an iterative balancing procedure in this particular case is likely to yield negligible differences in the results, as the median temporal discrepancy is just $1.62e^{-9}$, with the largest absolute discrepancy being $2.93e^{-5}$.

4 Empirical Results

In order to determine the most suitable method(s) of temporal disaggregation, given our dataset, we must select a set of test criteria for evaluating the performance of each method. Because of the diverse nature of the lines of business with respect to both magnitude and volatility, we are not only interested in the overall performance of each method, but how each performs on series with different temporal characteristics. Appendix B contains charts by selected LOB that show the performance of each method relative to the observed data for the highest and lowest volatility quintiles. For this paper, we use relative standard deviation, or standard deviation as a percent of the means for each observed series, as a measure of volatility, and create quintiles based on the relative standard deviation of each series, with Quintile 1 being the most volatile. Appendix Table B1 provides the composition of quintiles and relative standard deviation, by LOB. Consistent with Chen (2007), when analyzing the success of each method based on a given metric, we use the mean of each statistic across all 23 lines of business or, in the case of quintiles, across lines in a given quintile.

[14] Di Fonzo and Marini (2005) propose a method of benchmarking systems of series by solving simultaneously for temporal and contemporaneous constraints. Their choice of first differences matrix makes solving this system simpler, but is well known to impose transient movements at the beginning of the series (Cholette, 1984). Daalmans and Di Fonzo (2014) propose a specification of the Denton PFD and Causey-Trager models for solving a benchmarking problem with multiple series which simultaneously satisfies both temporal and contemporaneous constraints. Since we are disaggregating from annual observations, rather than benchmarking a given quarterly series to new annual constraints, we need to employ a two-stage procedure.

4.1 Accuracy of Level Prediction

We begin by examining the performance of each method based on statistics that measure the accuracy of each method with respect to the levels of the original series. Theil's (1958) inequality coefficient, U, which is an accuracy measure used in forecasting (Leuthold, 1975) used by Trabelsi and Hedhili (2005), is given by equation (8):

$$U = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}(p_n - a_n)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}(p_n^2)} + \sqrt{\frac{1}{N}\sum_{n=1}^{N}(a_n^2)}} \tag{8}$$

where p_n is the predicted value and a_n is the actual value in quarter n. The U statistic takes on a value between 0 and 1, where $U = 0$ indicates that the method used is a perfect predictor of the actual series (Leuthold, 1975; Trabelsi and Hedhili, 2005). Consistent with Trabelsi and Hedhili (2005), we also calculate the mean of the absolute differences between actual, a, and predicted values, p, for each line of business; we denote this as (M_{ap}).

Abeysinghe and Lee (1998) employed a different criterion, the root mean squared error as a percent of the mean (RMSE%) of the observed series, in which a lower value implies a more accurate prediction. Tables 1 and 2 display these three statistics for each quintile as well as the 23 lines of business overall. Appendix tables B2 and B3 provide these statistics for each individual line of business.

Table 1. Inequality Coefficient (U)and Absolute Mean Difference (M_{ap}), by Quintile

	Spline		D_{rel}		D_{cons}		GRP	
Quintile	U	M_{ap}	U	M_{ap}	U	M_{ap}	U	M_{ap}
1	0.0404	89.34	0.0393	82.98	0.0393	82.89	0.0393	81.10
2	0.0181	129.95	0.0181	127.78	0.0181	128.06	0.0181	127.55
3	0.0172	49.58	0.0172	47.04	0.0172	47.15	0.0172	47.09
4	0.0190	95.83	0.0190	96.00	0.0190	95.48	0.0190	95.78
5	0.0140	82.08	0.0140	68.39	0.0140	68.14	0.0140	69.21
All Lines	0.0221	89.32	0.0218	84.18	0.0218	84.06	0.0218	83.87

Note: Some values may appear equivalent due to rounding.

Table 2. Root Mean Squared Error %, by Quintile

Quintile	Spline	D_{rel}	D_{cons}	GRP
1	28.98	22.37	22.12	20.90
2	6.84	6.76	6.77	6.68
3	3.88	3.67	3.67	3.66
4	2.66	2.56	2.55	2.55
5	1.68	1.63	1.62	1.63
All Lines	9.11	7.59	7.53	7.25

Table 1 indicates the two Denton PFD methods, D_{rel} and D_{cons}, and the GRP method all perform similarly. D_{rel} marginally demonstrates the most accuracy based on the unrounded U statistic, while the GRP method is superior when looking at the M_{ap} and RMSE% measures; this is consistent with Abeysinghe and Lee (1998) who found that the RMSE% criterion suggested that a method using a related-series indicator was a better predictor of the actual series. All three measures suggest the spline method is inferior to the other methods. Looking at only the most volatile quintile (Quintile 1), we see D_{rel} performs slightly better than the other three methods based on the U statistic, while the GRP method outperforms the others with respect to the M_{ap} and RMSE% statistics; as before, the spline method is the least accurate based on these metrics.

4.2 Growth Rate Preservation

Each of the above criteria provides a measure of how well the methods perform based on the accuracy of their predicted *levels*. When working with movement and growth rate preservation models, we must also consider the ability of each method to properly estimate the growth rates of the target series. While many studies have focused on the preservation of the growth rate of the indicator series (Brown, 2012; Chen, 2007; Denton, 1971), due to our use of both a related series and a constant indicator in our Denton methods, we are interested in comparing the quarterly growth rates of the estimated series with those of the observed target series. To that end, we compare our predictions to the actual data rather than the indicator series to calculate our performance metrics.[15]

Following Di Fonzo and Marini (2005), we develop discrepancy statistics to analyze the extent to which the predicted series' quarterly rates of change deviate from those of the actual series. Table 3 displays the median revisions to quarterly rates of change, which are obtained by calculating the discrepancies between each actual and predicted quarterly growth rate. We then take the median of these discrepancies across all quarters for each LOB; to obtain quintile values, these values are averaged within each line's respective volatility quintile. Appendix Table B4 provides discrepancy statistics by line of business. Based on this metric, we find that the two modified Denton PFD methods outperform the other two methods, with respect to minimizing the deviation from the observed series growth rates.

Table 3. Revisions to Quarterly Rates of Change (RIPC), by Quintile

Quintile	Spline	D_{rel}	D_{cons}	GRP
1	1.01	1.13	1.11	1.38
2	0.65	0.37	0.41	0.93
3	0.29	0.27	0.27	0.61
4	0.48	0.41	0.42	0.73
5	0.13	0.11	0.12	0.50
All Lines	0.52	0.47	0.48	0.83

When looking only at Quintile 1, however, we find that the spline method has the fewest corrections to the rate of change, primarily due to significantly outperforming the others in the most volatile line of the system.

Chen (2007) also devised a method of analyzing discrepancies between growth rates of the predicted and indicator series, noting the importance of preserving short-term movements; we adapt these metrics to analyze discrepancies between our observed and predicted series and present the results by LOB in Tables B5 through B9 of Appendix B. One statistic used to evaluate the performance of growth rate preserving methods, C_P, measures the changes in period-to-period growth rates, according to equation (9):

$$C_P = \sum_{n=2}^{N} \frac{\left| \frac{\left(\frac{p_n}{p_{n-1}}\right)}{\left(\frac{a_n}{a_{n-1}}\right)} - 1 \right|}{N-1} \tag{9}$$

where a_n and p_n are the actual and predicted values in each quarter, respectively.

As indicated in Table 4, using this metric, the GRP method is most successful at achieving the short-term movement for the two most volatile quintiles, and the series overall; the two Denton PFD methods have superior performance in the remaining three quintiles, and the spline demonstrates the least successful performance. We note that the GRP method is not always as successful as noted in existing literature (Brown, 2012; Chen, 2012); this is likely due to the fact that the goal of the GRP method is to preserve the growth rate of an *indicator* series, whereas we are examining its performance in estimating the quarterly growth rate of the *actual* series. This finding does not fully align with that of Chen (2007) who found the Denton PFD method had the smallest value of C_P, though she notes that the performance was relatively similar to that of the Causey-Trager GRP method.

In addition to evaluating each method's ability to preserve short-term movements, Chen (2007) analyzes the ability of each method to minimize distortions between years and at the endpoints of a series. These factors become important when linking a newly benchmarked series to a previously benchmarked series, or when new benchmarking data becomes available; in the case of NAIC data, benchmarks become available on an annual

[15] Di Fonzo and Marini (2005a) and Trabelsi and Hedhili (2005) use predicted data in their performance metrics of movement preservation.

Table 4. Average Absolute Change in Period-to-Period Growth Rates (C_P), by Quintile

Quintile	Spline	D_{rel}	D_{cons}	GRP
1	0.1388	0.1383	0.1380	0.1351
2	0.0469	0.0451	0.0452	0.0445
3	0.0411	0.0400	0.0401	0.0400
4	0.0271	0.0265	0.0265	0.0265
5	0.0212	0.0208	0.0207	0.0208
All Lines	0.0559	0.0552	0.0551	0.0544

basis, suggesting distortions and discontinuities are a potential difficulty (Chen, 2007). Table 5 displays statistics on how each method performs with respect to distortions and series discontinuities. These statistics are based on average absolute differences in growth rates for given period-to-period percent changes, and are calculated for discontinuities at breaks between years (C_B) and in the middle of each year (C_M), as well as for distortions at the beginning and the end of each series, given by C_2 and C_T, respectively (Chen, 2007; Hood, 2005).

Table 5. Period-to-Period Distortions

Method	C_B	C_M	C_2	C_T
Spline	0.0516	0.0518	0.0998	0.1180
D_{rel}	0.0128	0.0114	0.0880	0.0951
D_{cons}	0.0007	0.0009	0.0885	0.0949
GRP	0.0038	0.0035	0.0895	0.0947

In each case, the spline method demonstrates a weaker performance, while the other three methods tend to differ only marginally. The Denton PFD with a constant indicator creates the least distortion between years (from the fourth quarter to the first quarter) as well as in the middle of each year (second quarter to third quarter). We find D_{rel} is most successful at preserving the growth rate from the first quarter to the second quarter of each series, and that the GRP method best preserves the growth rate for the final period of each series. When comparing the C_2 and C_T statistics across methods, we find the Denton PFD and GRP exhibit negligible differences, consistent with findings in Chen (2007) and Hood (2005).

4.3 Identification of Turning Points

Lastly, we conduct an analysis of turning points to determine how often each method correctly predicts a turning point in the actual data, how often it incorrectly predicts a turning point, and how often it does not predict a turning point where one exists (Trabelsi and Hedhili, 2005). Following Trabelsi and Hedhili (2005), we calculate Theil's (1958) statistics for turning points, or changes in the sign of the growth rate from period to period, using the Bry-Boschan algorithm (Bry and Boschan, 1971) to identify turning points in the actual and predicted series. In order to calculate turning points, we follow Berge and Jordà (2011), explicitly defining peaks (P_n) and troughs (T_n), where y_n denotes premiums in quarter, n, as:

$$\begin{aligned} P_n &= 1 \text{ if } \Delta ln(y_n) > 0 \text{ and} \Delta ln(y_{n+1}) < 0 \\ T_n &= 1 \text{ if } \Delta ln(y_n) < 0 \text{ and } \Delta ln(y_{n+1}) > 0 \end{aligned} \tag{10}$$

Theil (1958) constructs three variables to calculate turning point metrics, defined as (Theil, 1958; Trabelsi and Hedhili, 2005):

m_1 = number of turning points correctly predicted

m_2 = number of cases where turning points are incorrectly predicted

m_3 = number of cases where turning points are, incorrectly, not predicted

The turning point statistics ϕ_1 and ϕ_2 can then be calculated as shown in equations (11) and (12), respectively, and provide an indication of how well each method reproduces the true movement of the observed series:

$$\phi_1 = \frac{m_2}{(m_1 + m_2)} \tag{11}$$

$$\phi_2 = \frac{m_3}{(m_1 + m_3)} \tag{12}$$

Table 6 contains the results of the turning points analysis for each of the four methods, by quintile. In this analysis, ϕ_1 represents a Type I error, in which a turning point is identified where one does not truly exist (false positive); likewise, ϕ_2 represents a Type II error in which the model does not predict a turning point when one exists in the actual data (false negative).

Table 6. Turning Points, by Quintile, (ϕ_1 and ϕ_2)

	Spline		D_{rel}		D_{cons}		GRP	
Quintile	ϕ_1	ϕ_2	ϕ_1	ϕ_2	ϕ_1	ϕ_2	ϕ_1	ϕ_2
1	0.71	0.92	0.65	0.89	0.62	0.87	0.66	0.88
2	0.65	0.77	0.66	0.79	0.65	0.79	0.65	0.79
3	0.56	0.82	0.55	0.80	0.55	0.80	0.53	0.80
4	0.58	0.79	0.62	0.78	0.62	0.78	0.62	0.78
5	0.75	0.76	0.70	0.74	0.72	0.76	0.73	0.77
All Lines	0.65	0.81	0.64	0.80	0.63	0.80	0.64	0.81

Note: Some values may appear equivalent due to rounding.

While all four methods demonstrated equal performance for many individual lines of business, overall, the two Denton PFD methods outperformed the spline and GRP methods in most of the volatility quintiles. The D_{cons} method was slightly better than the D_{rel} specification, based on ϕ_1, suggesting it is marginally better at predicting turning points where they truly exist; D_{con} also proved slightly more successful in its ability to predict where turning points do not exist, as measured by ϕ_2. In the most volatile series, Quintile 1, the constant indicator Denton PFD method was most successful with regards to both measures.

5 Discussion and Conclusions

Similar to the existing literature, we not only find that one particular method is not consistently superior to the others, but that in many cases, those that outperform based on any given metric only do so marginally; this is apparent in the number of lines of business for which all four methods performed more or less equally well by most measures. Tables 7 and 8 summarize the relative ranking of each method based on each performance measure; the tables present the ranking of each method for all lines of business and for the most volatile quintile, respectively.

Table 7. Ranking of Method Performance for all Series

Method	RIPC	ϕ_1	ϕ_2	RMSE %	U	M_{ae}	C_P	C_B	C_M	C_2	C_T	Total
Spline	3	4	4	4	4	4	4	4	4	4	4	43
D_{rel}	1	2	2	3	1	2	3	3	3	1	3	24
D_{cons}	2	1	1	2	3	1	2	1	1	2	2	18
GRP	4	3	3	1	2	3	1	2	2	3	1	25

While there is mixed evidence, our results indicate that the modified Denton PFD method with a constant indicator is often most successful in estimating the temporal dynamics of the target series presented in this paper, based on its smallest total values in tables 7 and 8. The D_{rel} and the GRP methods typically perform similarly, but for this particular dataset, are often slightly inferior. When applied to insurance premiums, D_{cons} is most successful for both the 23 series overall and the most volatile quintile. While D_{rel} is slightly more successful than the GRP method overall, the nonlinear GRP method slightly outperforms D_{rel} in the most volatile quintile.

Table 8. Ranking of Method Performance for Qunitle 1

Method	RIPC	ϕ_1	ϕ_2	RMSE %	U	M_{ae}	C_P	C_B	C_M	C_2	C_T	Total
Spline	1	4	4	4	4	4	4	4	4	1	4	38
D_{rel}	3	2	2	3	1	3	3	3	3	2	3	28
D_{cons}	2	1	1	2	3	2	1	1	1	3	2	19
GRP	4	3	3	1	2	1	2	2	2	4	1	25

The success of D_{cons} with our dataset implies that temporal disaggregation extracts its most useful information from the annual data series, and does not rely heavily on the pattern series. While Quenneville et al. (2013) and Dagum and Cholette (2006) made a similar discovery using a natural cubic spline technique and a regression method, respectively, this finding still prompts one to look at the usefulness of imposing an aggregate trend on individual, often volatile, series; indeed, it is important to recognize that using a related series has the theoretical advantage of incorporating economic and statistical information into the estimated series (Pavía-Miralles, 2010). One advantage of our aggregate indicator is that it is obtained using the same methodology as the individual series we are estimating (Guerrero, 2003); however it is not known with certainty that each individual series follows the same data-generating process as that possessed by the aggregate trend. Because of the challenge posed by data availability, we chose to assess the relative performance of mathematical methods using the same indicator for each series, which also enables us to compare the merits of the methodologies, without potential confounding effects from individual indicator choices. Given appropriate data, it is possible improved indicators could bolster the performance of the other methods against the constant indicator, resulting in improved benchmarked estimates.

In addition to the possibility of using individual indicator series for each line of business, we also note that multiple statistical and regression-based methods exist for temporal disaggregation.[16] Many of these methods were developed to improve upon the mathematical methods by accounting for additional economic properties of a time series and by providing alternatives when a related series is not available (Chen, 2007). There is some evidence indicating regression methods are less successful in achieving short-term movement preservation, but do result in a smoother final series, which can be a desirable quality (Chen, 2007; Guerrero, 2003).

An alternate data-based procedure is the signal extraction method used by Guerrero and Nieto (1999) and Trabelsi and Hillmer (1990), which makes use of information in the preliminary series to estimate unobserved sub-annual data. Signal extraction procedures have the advantage of not requiring an indicator series, instead relying on the autoregressive features of the observed data to derive a pattern.[17] This method allows for meeting both temporal and contemporaneous constraints (Di Fonzo and Marini, 2005), and has also been shown to be a relative of the numerical approaches discussed in this paper. Trabelsi and Hillmer (1990) show that numerical benchmarking methods are in fact a special case of the signal extraction procedure. When compared with mathematical techniques, a signal extraction method was found to be successful on criteria such as turning points prediction, likely because they accounted for additional "stochastic properties of the aggregated and disaggregated series," and applied less smoothing to the final estimates (Trabelsi and Hedhili, 2005).

The priorities of the estimation process (e.g. smoothness of a series, frequency desired, and computational efficiency) can dictate the preferred method of temporal disaggregation, and decisions can be made based on the success of each method with respect to the most relevant accuracy measure. In national accounting, movement preservation and prediction of turning points are often a priority, suggesting either of the Denton PFD methods would be preferred for this use.[18] We have explored four mathematical methods, and found that a version of the modified Denton Proportional First Difference method, specifically that with a constant indicator, is most successful in disaggregating a series and achieving the least distortion from the actual values and movements of the sub-annual series, particularly in a national accounting setting. While this paper did not specifically assess the performance of statistical methods, many have been shown to produce results very similar to the mathematical methods (Chen, 2007; Di Fonzo and Marini, 2005), and warrant further investigation. Future research should make use of new and reliable data to continue evaluating mathematical and statistical methods, examine additional factors that may contribute to their ability to estimate the temporal dynamics of target series, and ultimately further the improvement of temporal disaggregation and benchmarking methods.

[16] See, for example, (Chen, 2007); (Chow and Lin, 1971); (Dagum and Cholette, 2006); (Guerrero, 2003); (Guerrero and Nieto, 1999); and (Trabelsi and Hedhili, 2005).

[17] While signal extraction makes use of an ARIMA process, Trabelsi and Hillmer (1990) suggest that a "pure ARIMA model" does not account for outliers and other effects.

[18] Indeed, Chen (2007) and Chen and Andrews (2008) note that BEA chose to adopt the Denton PFD method when both annual and a related-series indicator are available.

Appendix A Insurance Output Methodology

The Bureau of Economic Analysis (BEA) estimates annual output by line of business (LOB) for the property and casualty (P&C) insurance industry by applying the adaptive expectations model proposed by Chen and Fixler (2003). Their model seeks to capture the risk mitigation services provided by the insurance industry; using an adaptive expectations framework recognizes that firms base their operations in the current period on expectations derived from past experiences (Chen and Fixler, 2003).

Output in the current period is defined to be a function of premiums earned ($P_{i,t}$) and policyholder dividends paid ($D_{i,t}$) in the current period, as well as *expectations* for underwriting losses ($E[L_{i,t}]$) and investment gains (or losses) ($E[I_{i,t}]$) in the current period. That is, for each line of business (i) in year (t), output is defined to be:

$$Y_{i,t} = P_{i,t} - E[L_{i,t}] + E[I_{i,t}] - D_{i,t} \tag{A1}$$

where expected losses and investment gains are given as:

$$E[L_{i,t}] = \gamma L_{i,t-1} + (1-\gamma)E[L_{i,t-1}] \tag{A2a}$$

$$E[I_{i,t}] = \eta I_{i,t-1} + (1-\eta)E[I_{i,t-1}] \tag{A2b}$$

with $\gamma \in (0,1)$ and $\eta \in (0,1)$ defined as parameters representing the weight placed on observed losses and investment gains, respectively, in the prior period.

In order to align the quarterly output methodology with the annual methodology, BEA uses a lagged four-quarter moving average in the estimation of expected losses and investment gains.[19] That is, for each line of business (i) in quarter (q), output is defined to be:

$$Y_{i,q} = P_{i,q} - E[L_{i,q}] + E[I_{i,q}] \tag{A3}$$

where expected losses and investment gains are given as:

$$E[L_{i,q}] = \frac{\gamma}{4}\sum_{j=1}^{4} L_{i,q-j} + \frac{(1-\gamma)}{4}\sum_{j=1}^{4} E[L_{i,q-j}] \tag{A4a}$$

$$E[I_{i,q}] = \frac{\eta}{4}\sum_{j=1}^{4} I_{i,q-j} + \frac{(1-\eta)}{4}\sum_{j=1}^{4} E[I_{i,q-j}] \tag{A4b}$$

with parameters γ and η defined as before.

For computational efficiency, it can be shown that this solution can be approximated by a lagged exponential moving average, with successively smaller weights placed on observations farther back in time.

[19] Policyholder dividends are a relatively small component of output. Since reliable and timely data for policyholder dividends are not available on a quarterly basis, they are excluded from the quarterly model.

Appendix B Selected Tables and Charts by Line of Business

In this Appendix we present selected tables from Section 5, providing detailed statistics by line of business. All tables are organized by ascending order of volatility. Following tables B1 - B9, we provide charts for Quintiles 1 and 5 (the most and least volatile quintiles, respectively). Each chart displays the level of premiums for each method, as well as the actual series for the entire time series, 2002 - 2012. The top right quandrant contains a chart with the ratios of the revised series to actual series, for each method, while the bottom left quandrant shows the average annual growth for the observed data and the predictions produced by each method. Finally, the bottom right quadrant indicates the extent to which each method over- or under-predicts the actual data.

Table B1. Relative Standard Deviation

Line of Business	Rel. St. Dev.
Auto Physical Damage	0.028
Commercial Multiple Peril	0.058
Private Passenger Auto Liability	0.060
Medical Malpractice	0.082
Boiler and Machinery	0.083
Commercial Auto Liability	0.088
Fidelity	0.091
Product & Other Liability	0.094
Workers' Compensation	0.112
Surety	0.116
Inland Marine	0.118
Ocean Marine	0.121
Fire	0.143
Earthquake	0.155
Farmowners' Multiple Peril	0.162
Homeowners' Multiple Peril	0.170
Aggregate Write-ins for Other Lines	0.200
Aircraft	0.249
Other Accident & Health	0.258
Allied Lines	0.309
Burglary & Theft	0.324
Group Accident & Health	0.403
Credit Accident & Health	1.012

Table B2. Inequality Coefficient (U)and Absolute Mean Difference M_{ap})

	Spline		D_{rel}		D_{cons}		GRP	
Line of Business	U	M_{ap}	U	M_{ap}	U	M_{ap}	U	M_{ap}
AutoDam	0.0052	119.9219	0.0052	85.8767	0.0052	84.9579	0.0052	88.1998
ComMP	0.0077	79.4255	0.0076	68.3244	0.0076	68.3912	0.0076	68.1569
PrivAuto	0.0045	161.6504	0.0045	137.9858	0.0045	137.7438	0.0045	139.8924
Medmal	0.0134	41.8029	0.0134	41.7992	0.0134	41.6069	0.0134	41.6642
Boiler	0.0392	7.5892	0.0391	7.9833	0.0391	8.0169	0.0391	8.1217
CommAuto	0.0098	77.3232	0.0098	68.5141	0.0098	68.9175	0.0098	69.0450
Fidelity	0.0403	7.8556	0.0403	7.7562	0.0403	7.6755	0.0403	7.6918
Product	0.0061	228.6767	0.0061	246.9826	0.0061	244.1208	0.0061	246.3042
Workers	0.0064	152.7867	0.0064	145.3738	0.0064	145.2990	0.0064	144.6085
Surety	0.0202	29.1212	0.0202	27.2133	0.0202	27.1752	0.0202	27.1262
Inland	0.0121	51.5435	0.0121	46.0002	0.0121	46.0756	0.0121	46.1489
Ocean	0.0234	34.6681	0.0233	34.7322	0.0233	34.7868	0.0233	34.8166
Fire	0.0130	82.9860	0.0130	80.2177	0.0130	80.5475	0.0130	80.2829
Earth	0.0323	12.5069	0.0322	11.3548	0.0322	11.3863	0.0322	11.2731
Farm	0.0272	10.6374	0.0271	10.1211	0.0271	10.1077	0.0271	10.0959
Home	0.0056	113.9585	0.0056	85.0710	0.0056	86.1424	0.0056	85.5341
AggOther	0.0122	351.9669	0.0121	376.7520	0.0121	376.7633	0.0121	376.5623
Aircraft	0.0274	43.2193	0.0274	39.1574	0.0274	39.2304	0.0274	38.0066
OtherAH	0.0231	79.3347	0.0231	79.1541	0.0231	78.7359	0.0231	78.7735
Allied	0.0098	251.6823	0.0098	221.3190	0.0098	221.2343	0.0098	221.1119
Burglary	0.1035	6.5293	0.1029	6.6783	0.1029	6.6989	0.1029	6.4096
GroupAH	0.0175	68.7878	0.0175	70.5796	0.0175	71.8570	0.0175	68.0453
CreditAH	0.0479	40.3589	0.0430	37.1835	0.0430	35.9333	0.0430	31.1425

Table B3. Root Mean Squared Error %

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.83	0.69	0.69	0.70
ComMP	1.20	1.07	1.07	1.07
PrivAuto	0.83	0.74	0.74	0.75
Medmal	2.36	2.40	2.39	2.39
Boiler	3.19	3.22	3.24	3.26
CommAuto	1.89	1.64	1.65	1.65
Fidelity	3.58	3.61	3.59	3.60
Product	2.82	2.86	2.85	2.85
Workers	1.61	1.57	1.57	1.56
Surety	3.42	2.91	2.92	2.91
Inland	2.21	2.08	2.08	2.10
Ocean	5.16	5.06	5.06	5.07
Fire	4.73	4.61	4.63	4.58
Earth	3.40	3.11	3.11	3.10
Farm	2.03	1.94	1.94	1.93
Home	1.00	0.83	0.83	0.83
AggOther	14.57	15.79	15.81	15.79
Aircraft	9.75	8.49	8.52	8.18
OtherAH	18.12	17.70	17.63	17.55
Allied	6.89	6.13	6.16	6.15
Burglary	26.47	23.82	23.86	23.51
GroupAH	7.89	8.04	8.11	7.87
CreditAH	85.54	56.14	54.85	49.42

Table B4. Revisions to Quarterly Rates of Change

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	-0.14	-0.05	-0.04	0.00
ComMP	0.02	0.08	0.06	0.90
PrivAuto	-0.05	-0.08	-0.08	0.50
Medmal	0.26	0.14	0.19	-0.38
Boiler	-0.19	-0.217	-0.224	0.71
CommAuto	-0.24	-0.24	-0.25	0.24
Fidelity	-0.61	-0.80	-0.80	0.84
Product	-0.33	-0.20	-0.26	0.82
Workers	0.09	0.27	0.25	0.55
Surety	-0.19	-0.27	-0.24	0.10
Inland	0.29	0.24	0.24	0.63
Ocean	-0.35	0.28	0.28	0.86
Fire	-0.32	0.31	0.34	0.86
Earth	1.12	0.56	0.55	1.19
Farm	-0.57	-0.43	-0.44	1.71
Home	0.35	0.02	0.04	1.32
AggOther	-0.57	-0.60	-0.65	-0.47
Aircraft	1.11	0.45	0.49	-0.21
OtherAH	-1.08	-0.79	-0.75	-1.08
Allied	-1.70	-0.92	-0.89	2.44
Burglary	1.16	0.81	0.83	1.13
GroupAH	-0.81	-1.33	-1.32	0.05
CreditAH	0.30	-1.78	-1.78	-2.19

Table B5. Average Absolute Change in Period-to Period Growth Rates (C_P)

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.0082	0.0021	0.0002	0.0004
ComMP	0.0123	0.0024	0.0006	0.0008
PrivAuto	0.0098	0.0016	0.0001	0.0003
Medmal	0.0246	0.0030	0.0012	0.0013
Boiler	0.0359	0.0034	0.0004	0.0009
CommAuto	0.0168	0.0021	0.0006	0.0008
Fidelity	0.0404	0.0040	0.0008	0.0008
Product	0.0246	0.0048	0.0005	0.0005
Workers	0.0194	0.0019	0.0006	0.0008
Surety	0.0370	0.0068	0.0029	0.0028
Inland	0.0246	0.0048	0.0021	0.0022
Ocean	0.0572	0.0060	0.0003	0.0005
Fire	0.0459	0.0042	0.0008	0.0014
Earth	0.0341	0.0058	0.0023	0.0028
Farm	0.0266	0.0032	0.0006	0.0008
Home	0.0093	0.0018	0.0002	0.0003
AggOther	0.0533	0.0062	0.0016	0.0021
Aircraft	0.0969	0.0141	0.0051	0.0099
OtherAH	0.1335	0.0376	0.0265	0.0295
Allied	0.0758	0.0095	0.0011	0.0017
Burglary	0.2071	0.0494	0.0057	0.0205
GroupAH	0.0763	0.0121	0.0020	0.0092
CreditAH	0.1478	0.1190	0.0074	0.0330

Table B6. Distortions at Breaks between Years (C_B)

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.0082	0.0020	0.0001	0.0003
ComMP	0.0109	0.0013	0.0001	0.0002
PrivAuto	0.0093	0.0013	0.0001	0.0003
Medmal	0.0233	0.0025	0.0002	0.0003
Boiler	0.0461	0.0033	0.0003	0.0007
CommAuto	0.0168	0.0016	0.0001	0.0003
Fidelity	0.0370	0.0032	0.0003	0.0005
Product	0.0387	0.0072	0.0003	0.0003
Workers	0.0225	0.0023	0.0002	0.0004
Surety	0.0441	0.0048	0.0003	0.0004
Inland	0.0278	0.0027	0.0002	0.0004
Ocean	0.0769	0.0051	0.0002	0.0005
Fire	0.0811	0.0046	0.0003	0.0011
Earth	0.0323	0.0041	0.0001	0.0005
Farm	0.0279	0.0020	0.0001	0.0004
Home	0.0083	0.0015	0.0001	0.0003
AggOther	0.0627	0.0094	0.0003	0.0010
Aircraft	0.0859	0.0086	0.0008	0.0073
Allied	0.0869	0.0088	0.0005	0.0015
Burglary	0.1474	0.0755	0.0016	0.0234
GroupAH	0.1159	0.0120	0.0014	0.0098
CreditAH	0.1229	0.1175	0.0072	0.0340
OtherAH	0.0541	0.0133	0.0011	0.0032

Table B7. Average Absolute Change in Growth Rates, Middle of the Year (C_M)

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.0080	0.0023	0.0001	0.0003
ComMP	0.0089	0.0029	0.0002	0.0003
PrivAuto	0.0106	0.0025	0.0001	0.0003
Medmal	0.0192	0.0028	0.0003	0.0004
Boiler	0.0287	0.0041	0.0003	0.0010
CommAuto	0.0169	0.0020	0.0001	0.0003
Fidelity	0.0515	0.0047	0.0005	0.0004
Product	0.0106	0.0049	0.0005	0.0003
Workers	0.0153	0.0012	0.0002	0.0004
Surety	0.0258	0.0064	0.0006	0.0004
Inland	0.0220	0.0035	0.0003	0.0004
Ocean	0.0266	0.0070	0.0004	0.0004
Fire	0.0240	0.0042	0.0005	0.0013
Earth	0.0299	0.0055	0.0004	0.0011
Farm	0.0206	0.0037	0.0002	0.0004
Home	0.0082	0.0021	0.0002	0.0004
AggOther	0.0530	0.0039	0.0006	0.0009
Aircraft	0.1346	0.0135	0.0008	0.0062
OtherAH	0.1247	0.0161	0.0019	0.0052
Allied	0.0603	0.0095	0.0008	0.0010
Burglary	0.2446	0.0380	0.0032	0.0174
GroupAH	0.0459	0.0119	0.0016	0.0088
CreditAH	0.2004	0.1086	0.0066	0.0318

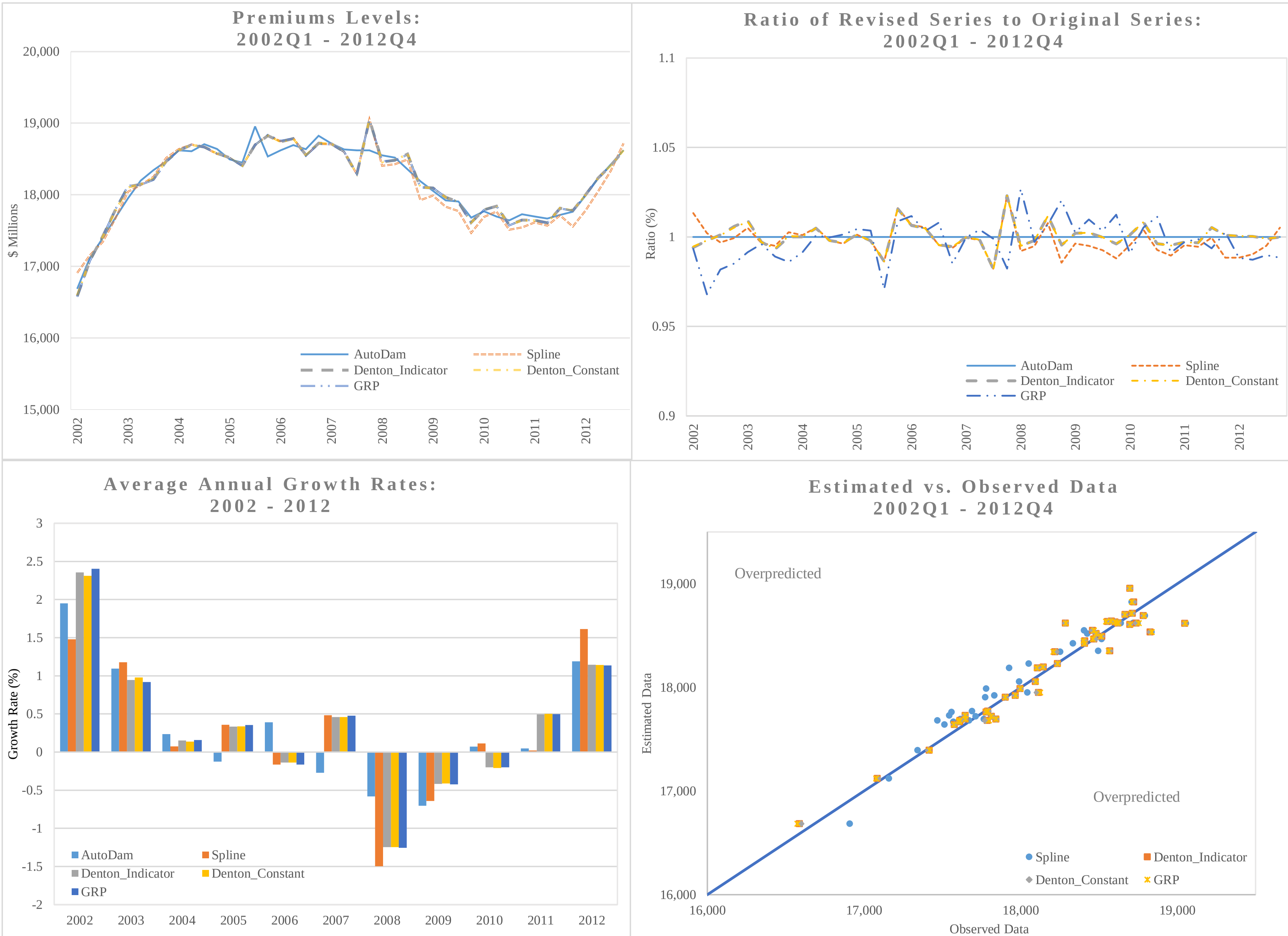
Table B8. Absolute Change in Period-to-Period Growth Rate in the Second Period (C_2)

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.0114	0.0041	0.0038	0.0042
ComMP	0.0142	0.0226	0.0225	0.0228
PrivAuto	0.0075	0.0028	0.0028	0.0025
Medmal	0.0598	0.0446	0.0454	0.0451
Boiler	0.0087	0.0068	0.0073	0.0066
CommAuto	0.0379	0.0206	0.0208	0.0204
Fidelity	0.0450	0.0173	0.0184	0.0178
Product	0.0467	0.0073	0.0059	0.0068
Workers	0.0235	0.0199	0.0203	0.0204
Surety	0.1781	0.1081	0.1093	0.1089
Inland	0.0670	0.0845	0.0841	0.0845
Ocean	0.0093	0.0029	0.0028	0.0032
Fire	0.0139	0.0161	0.0164	0.0166
Earth	0.1302	0.0890	0.0892	0.0888
Farm	0.0191	0.0211	0.0209	0.0207
Home	0.0055	0.0007	0.0003	0.0003
AggOther	0.0757	0.0491	0.0493	0.0496
Aircraft	0.2832	0.1924	0.1924	0.1932
OtherAH	1.1971	1.0786	1.0822	1.0852
Allied	0.0403	0.0216	0.0211	0.0211
Burglary	0.0011	0.1559	0.1527	0.1565
GroupAH	0.0144	0.0256	0.0281	0.0274
CreditAH	0.0064	0.0332	0.0401	0.0568

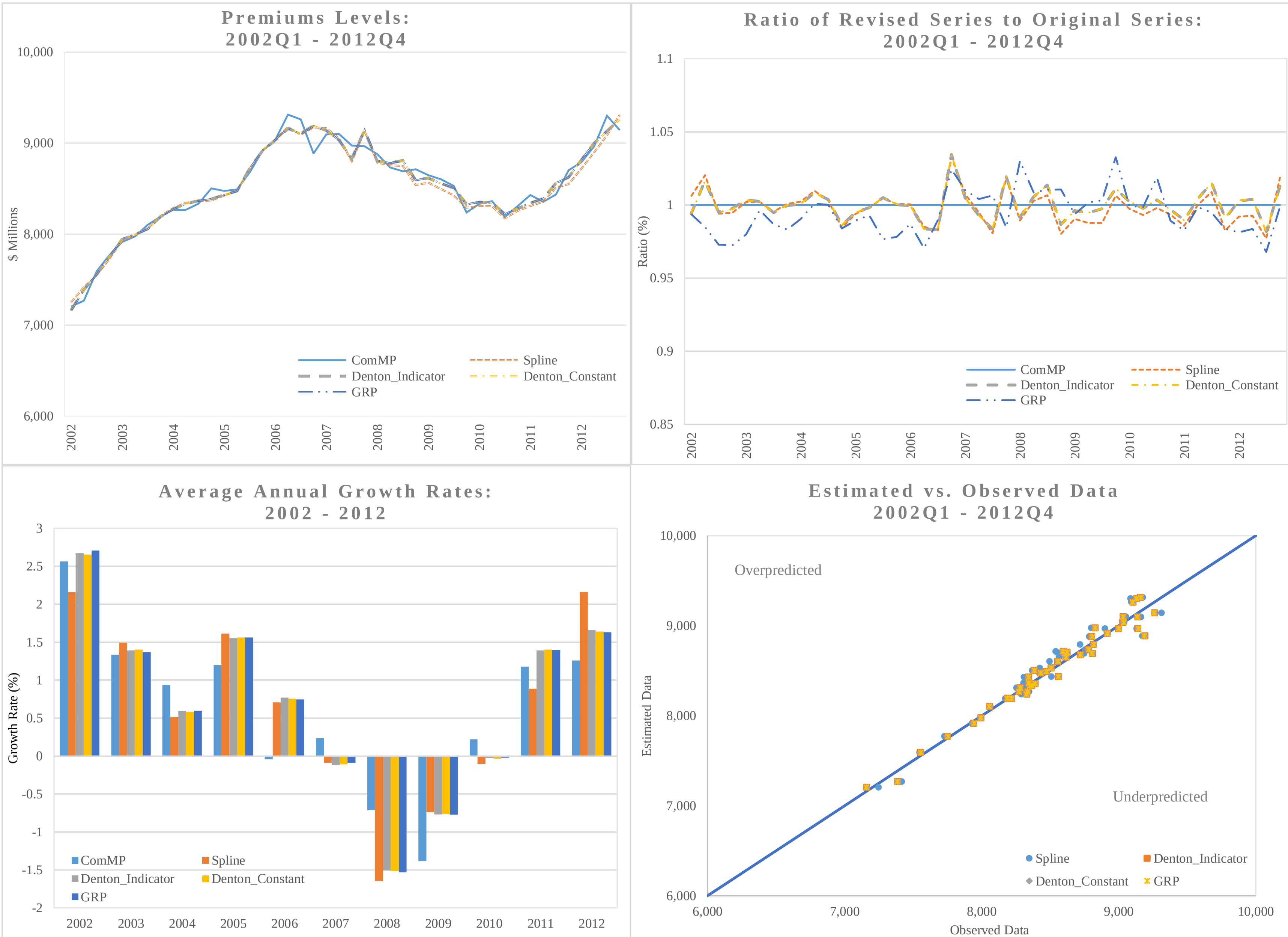
Table B9. Absolute Change in Last Period Growth Rate (C_t)

Line of Business	Spline	D_{rel}	D_{cons}	GRP
AutoDam	0.0051	0.0146	0.0146	0.0147
ComMP	0.0160	0.0052	0.0050	0.0049
PrivAuto	0.0191	0.0244	0.0244	0.0245
Medmal	0.0116	0.0129	0.0132	0.0136
Boiler	0.0435	0.0226	0.0227	0.0228
CommAuto	0.0375	0.0408	0.0407	0.0408
Fidelity	0.0308	0.0148	0.0145	0.0144
Product	0.0186	0.0340	0.0342	0.0343
Workers	0.0032	0.0010	0.0008	0.0008
Surety	0.1279	0.1272	0.1268	0.1265
Inland	0.0957	0.0728	0.0724	0.0726
Ocean	0.1010	0.0265	0.0259	0.0252
Fire	0.0276	0.0106	0.0104	0.0102
Earth	0.1014	0.1073	0.1074	0.1075
Farm	0.0190	0.0397	0.0399	0.0400
Home	0.0127	0.0229	0.0229	0.0231
AggOther	0.0137	0.0173	0.0179	0.0190
Aircraft	0.1593	0.1751	0.1749	0.1748
OtherAH	1.1063	1.0841	1.0835	1.0825
Allied	0.1641	0.0547	0.0540	0.0537
Burglary	0.1519	0.1523	0.1523	0.1520
GroupAH	0.0711	0.0334	0.0331	0.0328
CreditAH	0.2727	0.0148	0.0140	0.0122

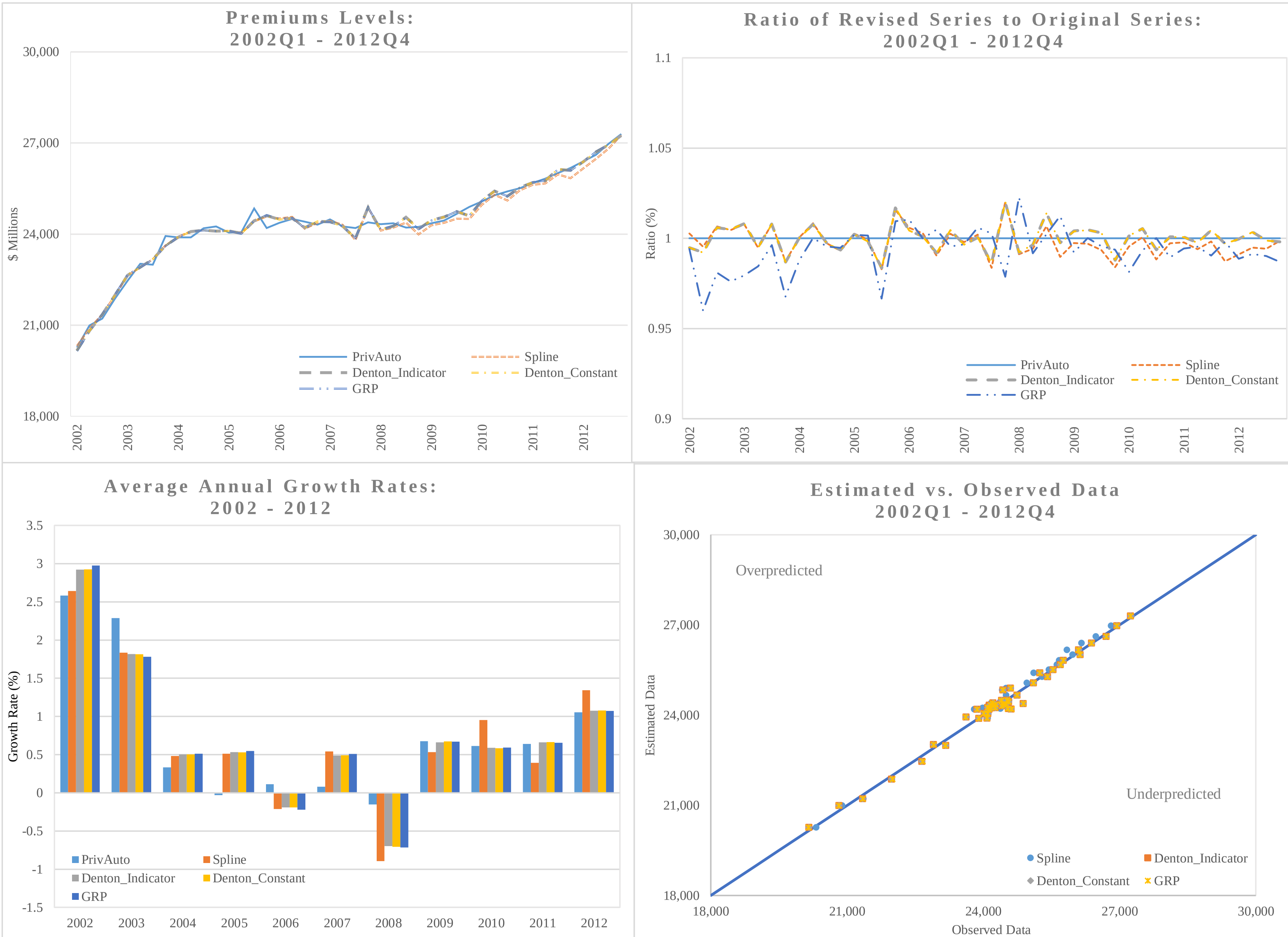
AUTO DAMAGE PREMIUMS, BY METHOD: 2002Q1 – 2012Q4



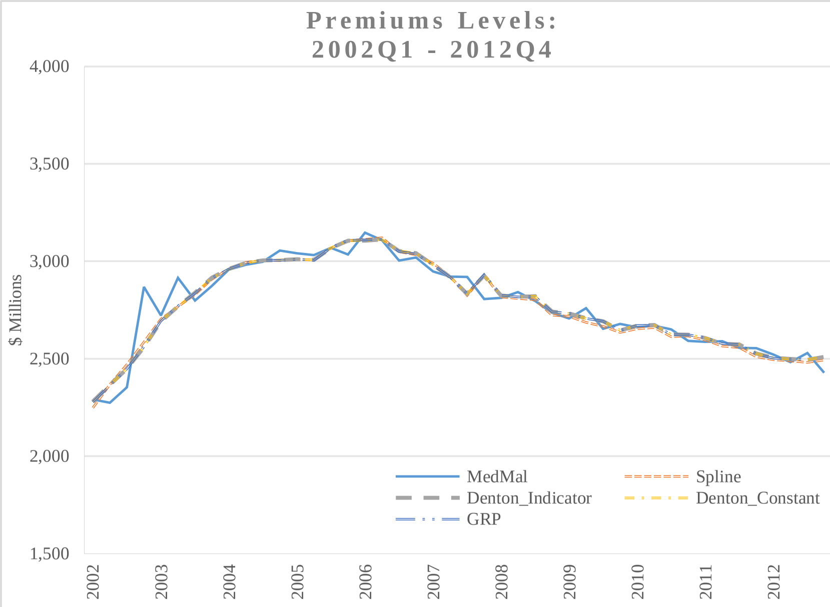
Commercial Multiple Peril Premiums, by Method: 2002Q1 – 2012Q4

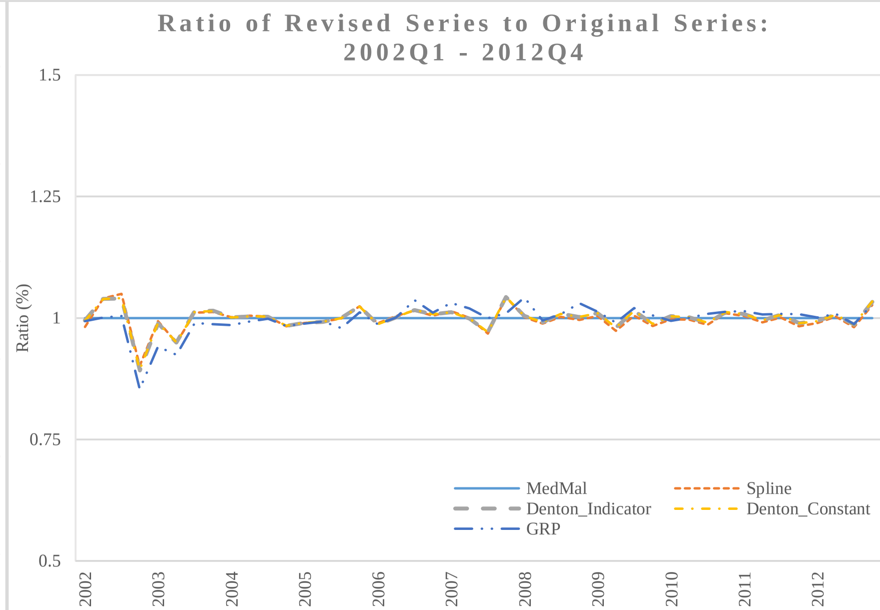


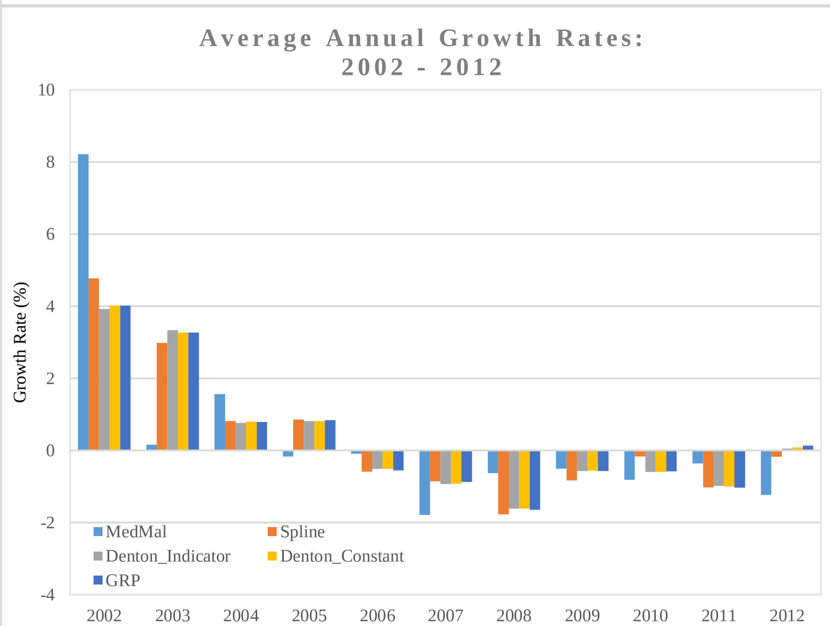
Private Auto Damage Premiums, by Method: 2002Q1 – 2012Q4

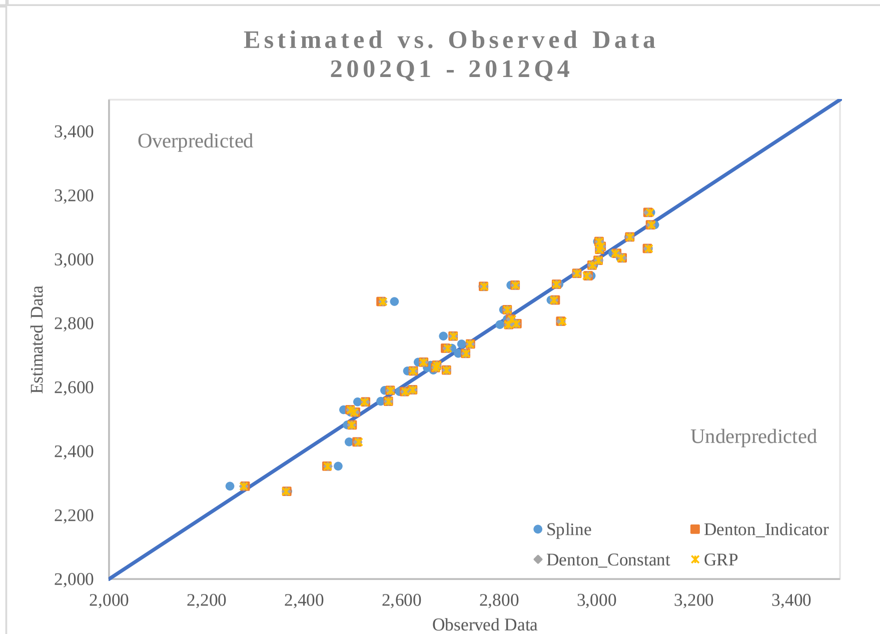


Medical Malpractice Premiums, by Method: 2002Q1 – 2012Q4

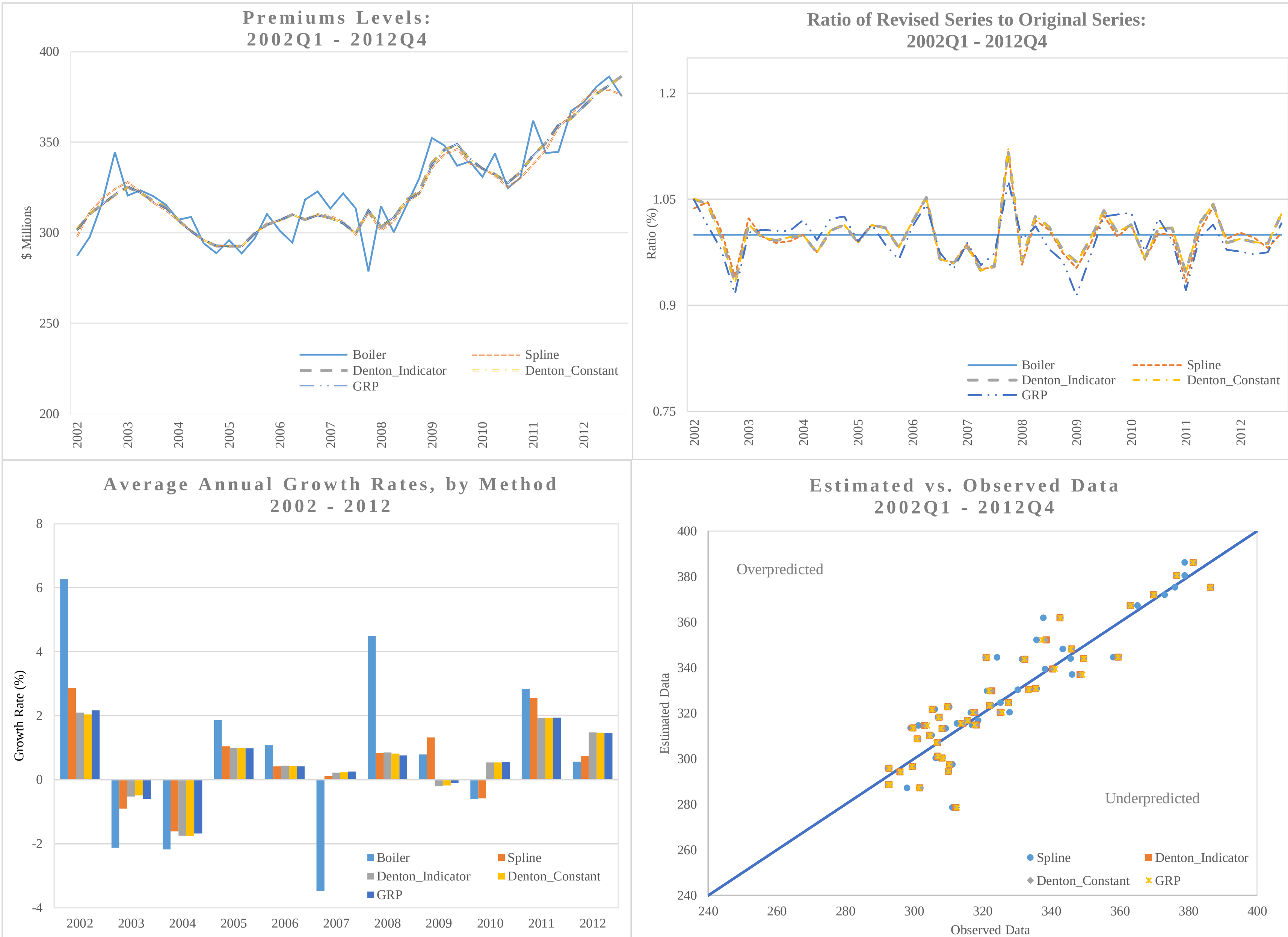




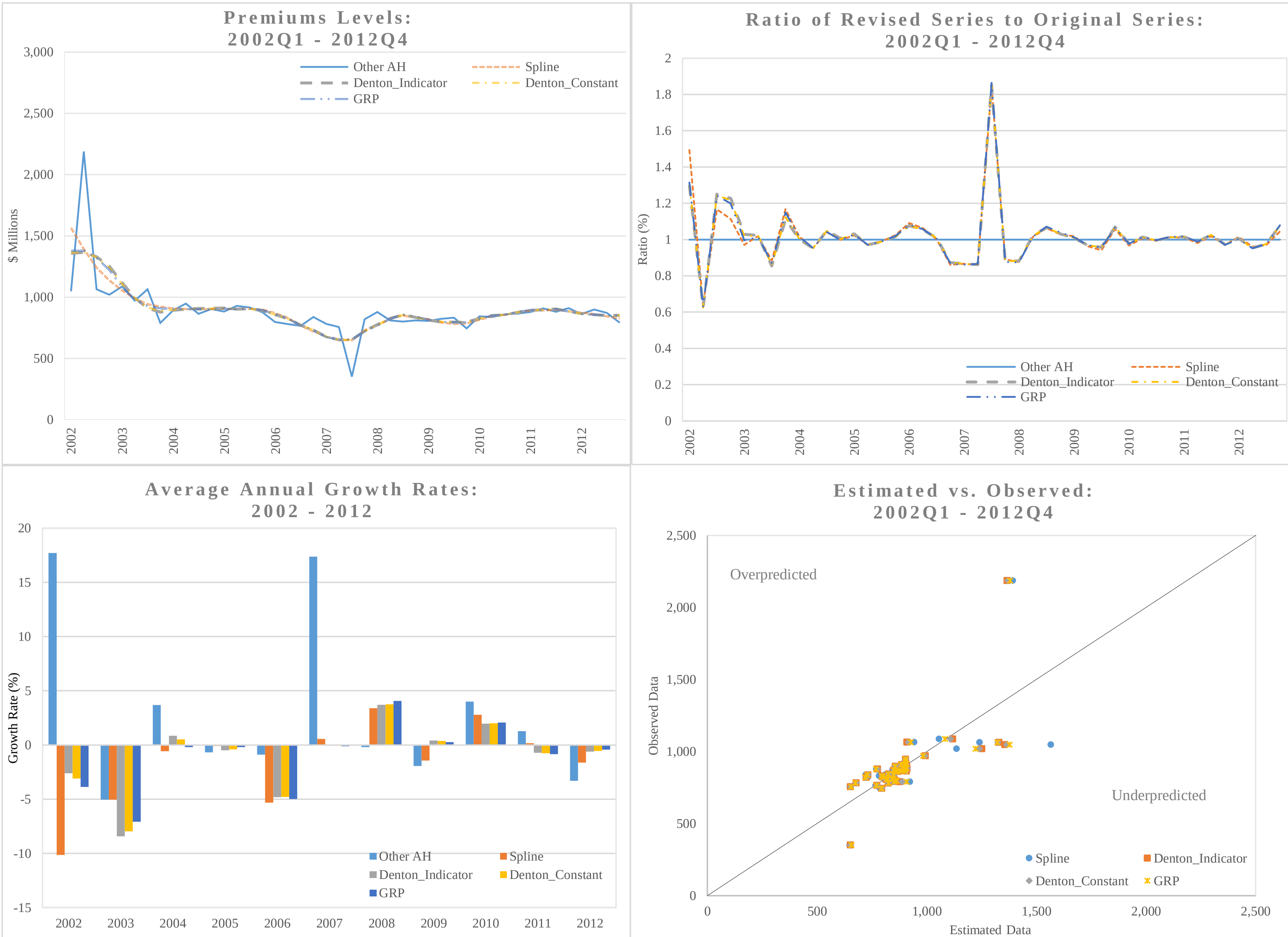




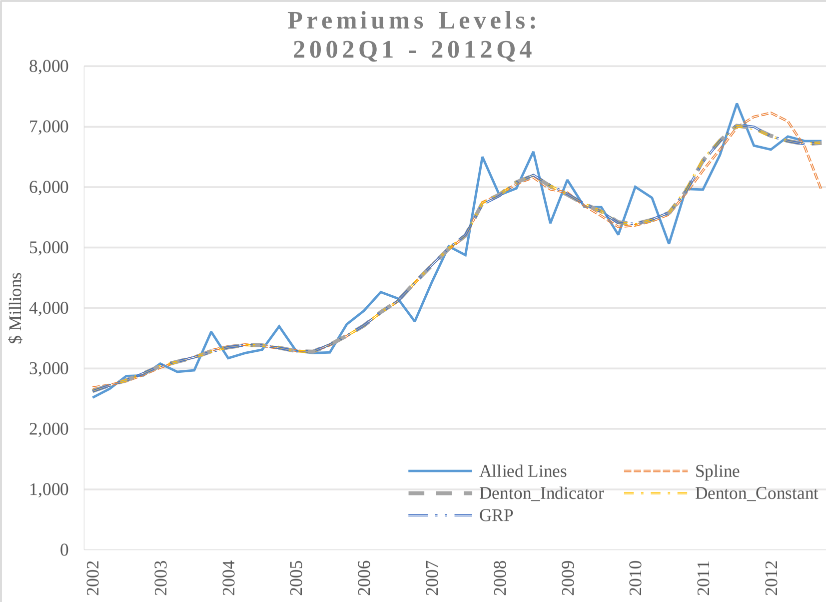
Boiler and Machinery Premiums, by Method: 2002Q1 – 2012Q4

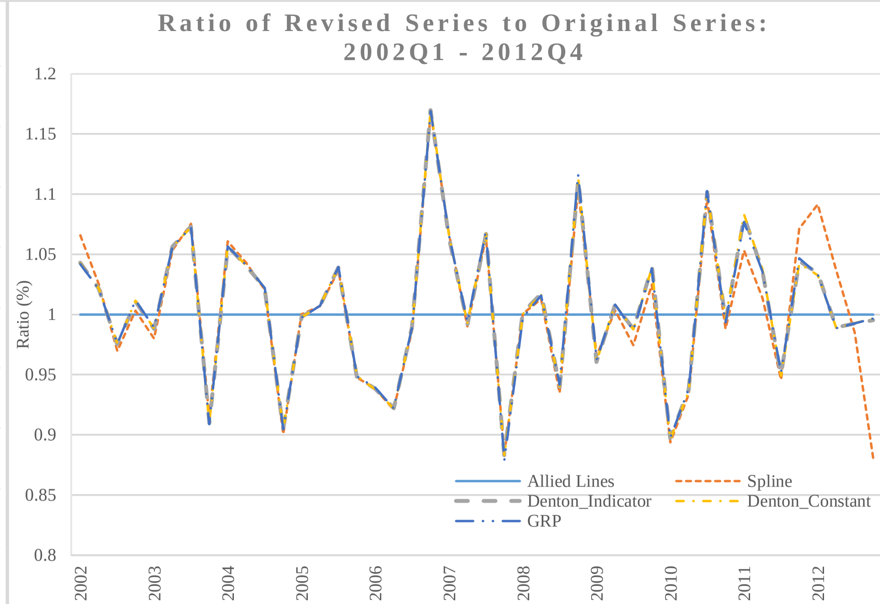


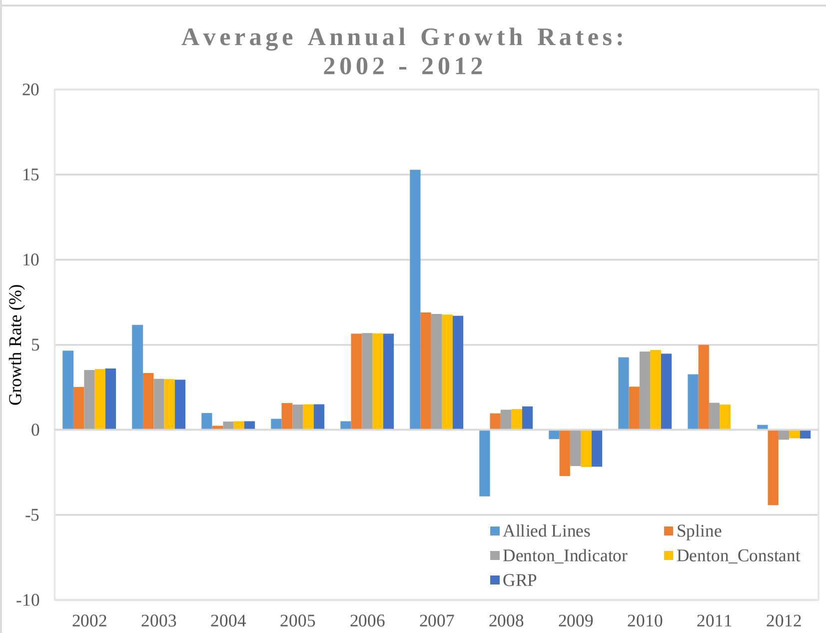
Other Accident & Health Premiums, by Method: 2002Q1: 2012Q4

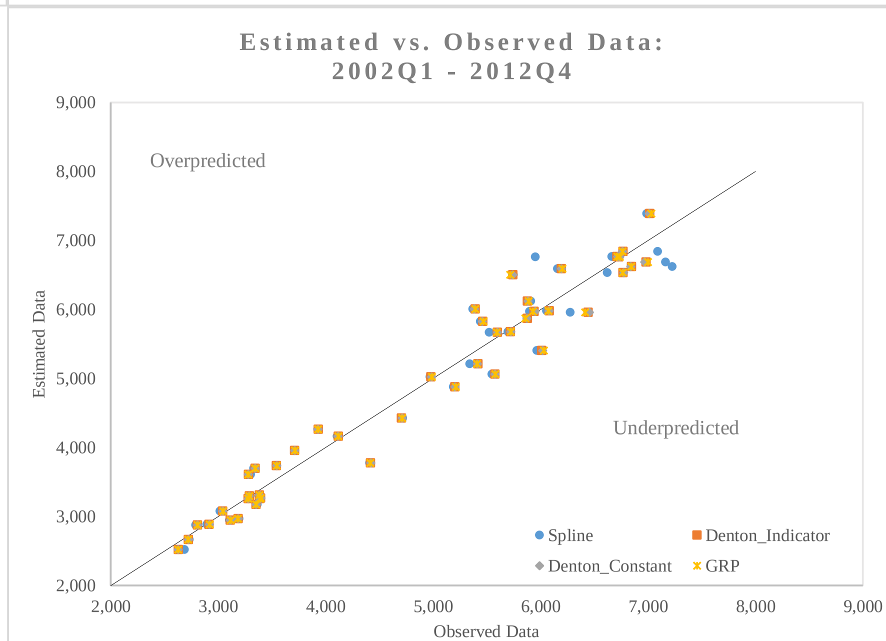


Allied Lines Premiums, by Method: 2002Q1-2012Q4

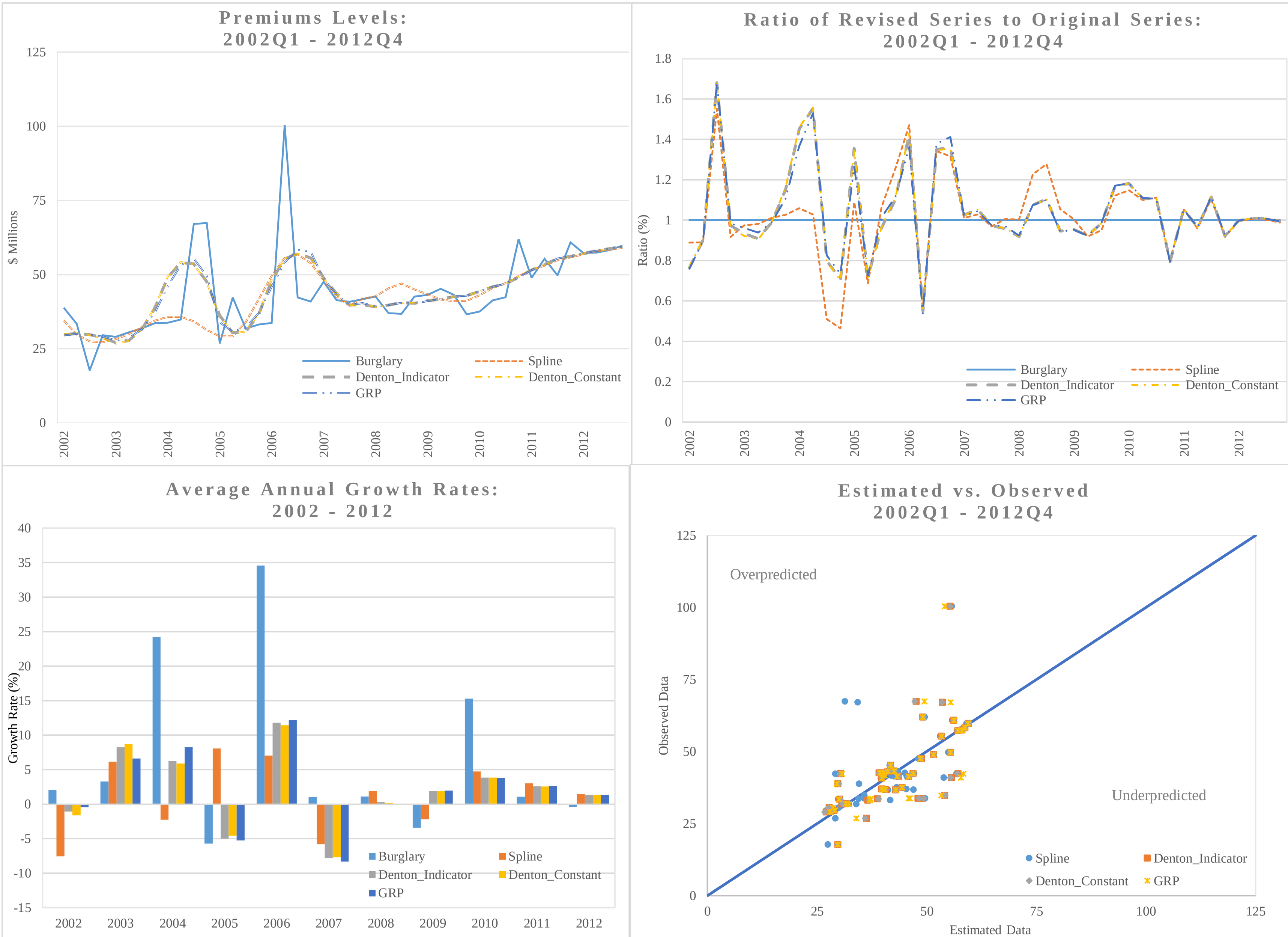




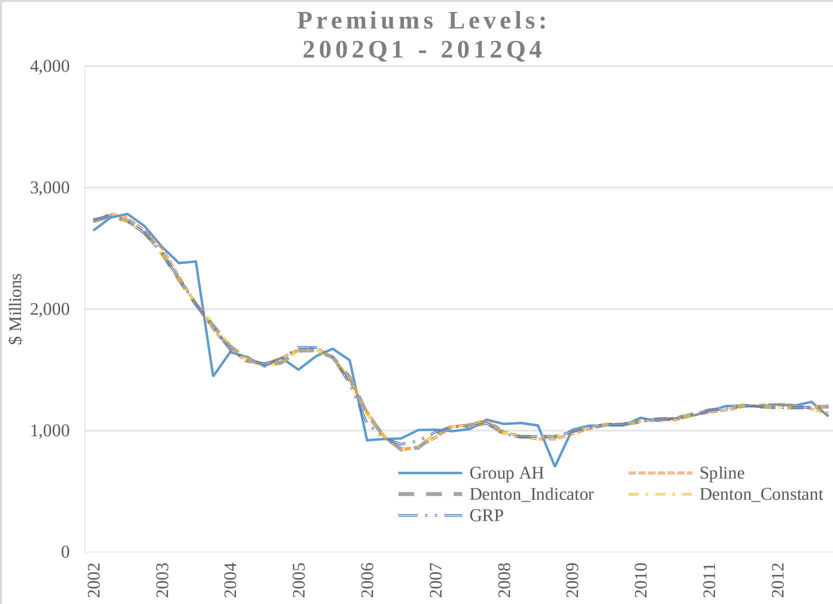


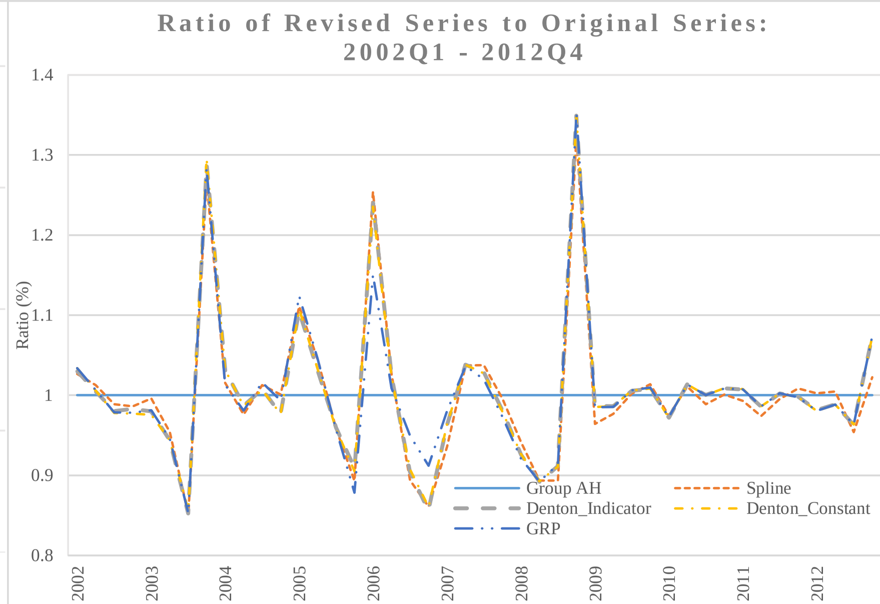


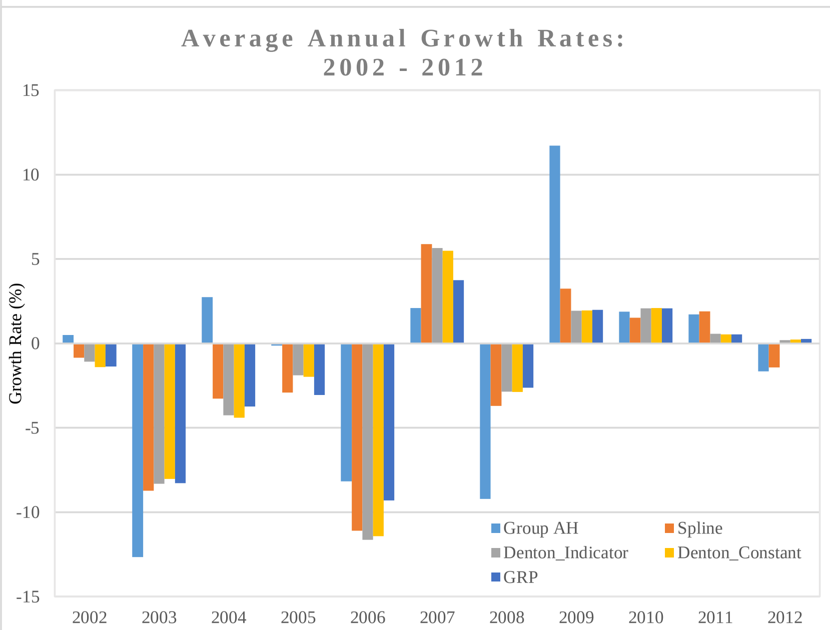
BURGLARY & THEFT PREMIUMS, BY METHOD: 2002Q1 – 2012Q4

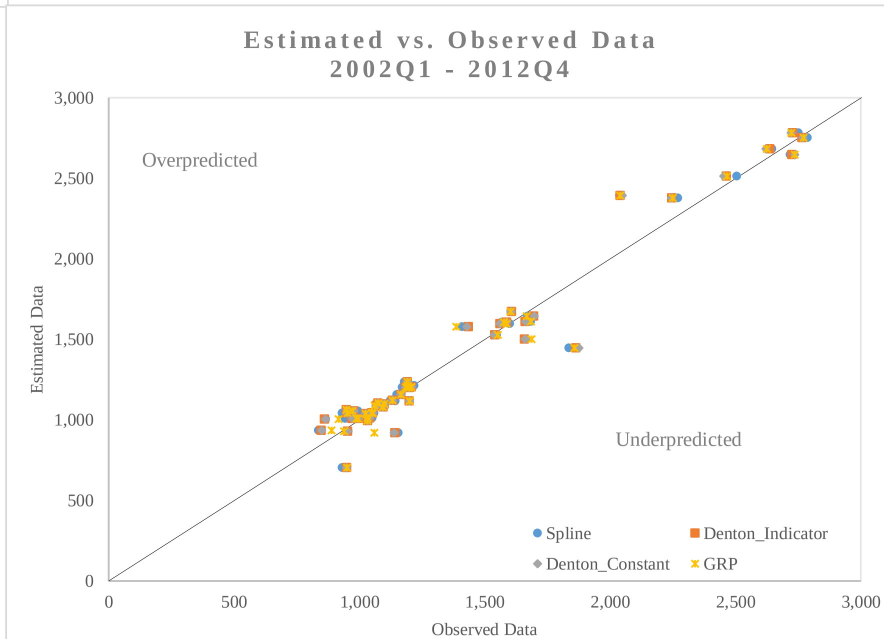


GROUP ACCIDENT & HEALTH PREMIUMS, BY METHOD: 2002Q1 – 2012Q4

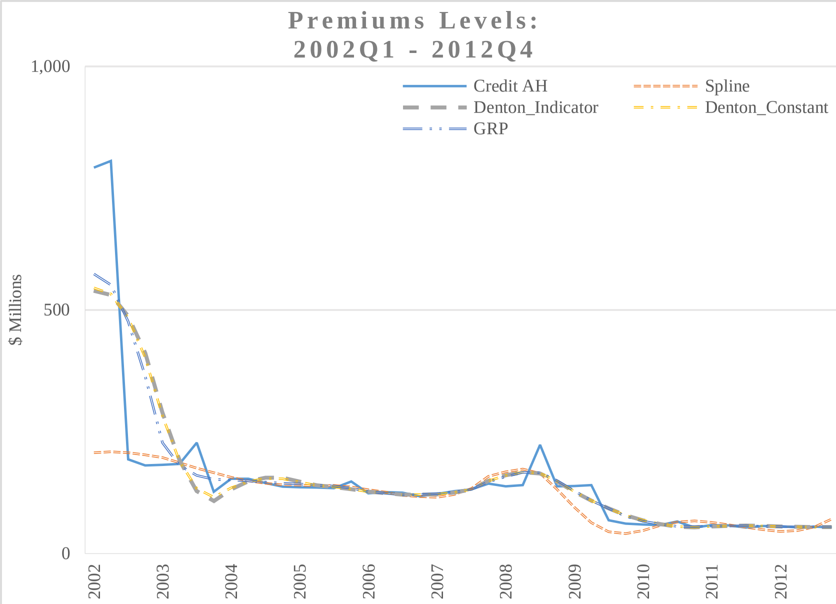


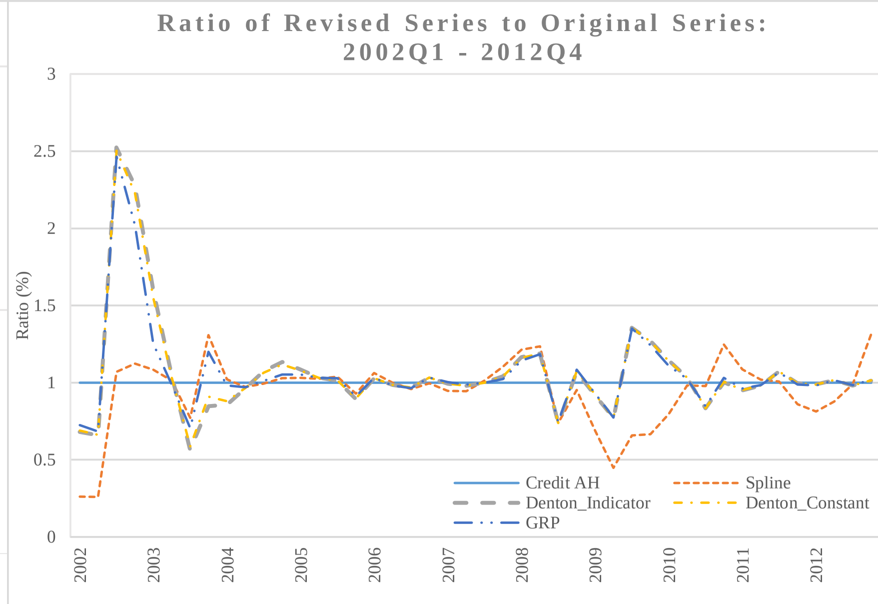


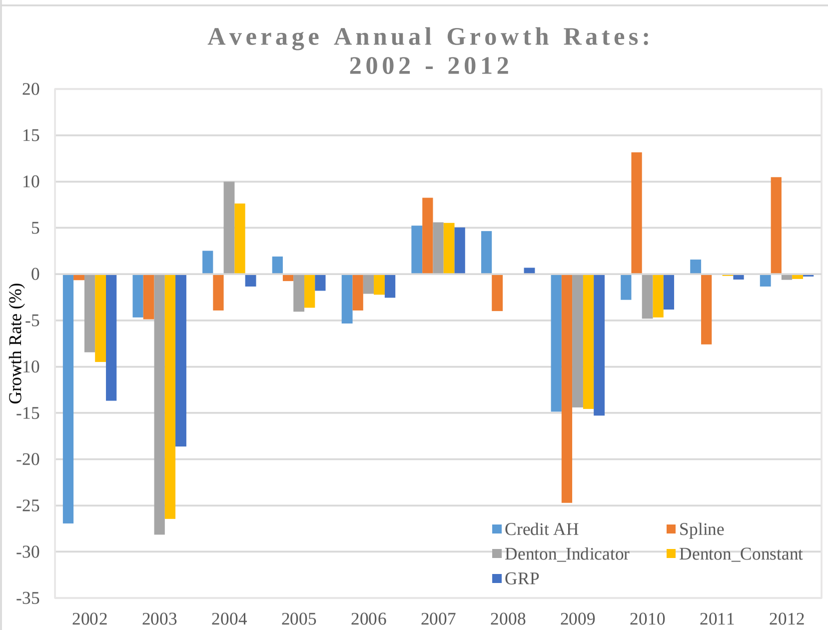


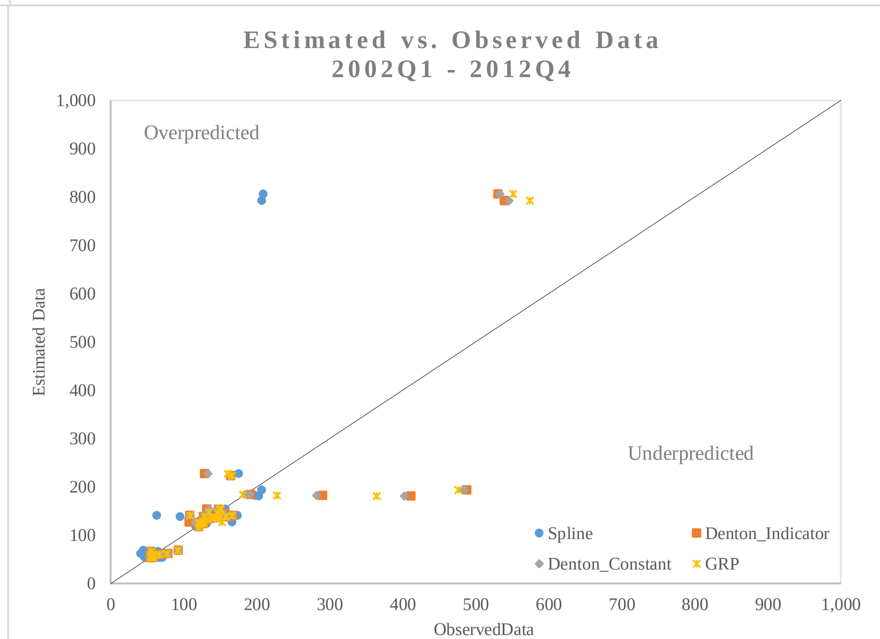


Credit Accident & Health Premiums, by Method: 2002Q1: 2012Q4









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