Structural Estimates of Depreciation
from Wholesale Auctions of Used Ford Windstar Vans

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Version of July 17, 2018

Abstract

This paper estimates the depreciation of over twenty varieties of Ford Windstar passenger vans, using several hundred thousand observations from a deep collection of wholesale auction transactions. The data permit the estimation of individual vehicle ages apart from differences between resale- and model-years, so that age-, date-, and vintage- effects are all identifiable. Depreciation is modeled in two parts, estimated separately: obsolescence, wherein new model-years depress the resale values of older cohorts uniformly; and ordinary wear-and-tear, which reduces the resale values of individual vehicles in a nonuniform manner summarized by different individual-level service-lives. Obsolescence (and inflation) are estimated first, in a regression setting of third-order functions of age and miles to hold individual effects constant, allowing varietal and quarterly time dummies to trace each variety's counterfactual "as-if-new" price through time in a way that upholds the useful fiction of a Hicksian aggregate. The estimated as-if-new prices then anchor individual-level resale-price profiles, which are embedded in implicit service-life densities that vary over time and age. Quarterly moments of prices and miles are then matched by adjusting parameters for wear-and-tear.

As-if-new prices fall fast (over 10 percent a year, on average) — faster than the observed reductions of actual nearly-new prices, so there is substantial obsolescence. Individual-level service-flows decline in a manner similar to one long assumed by the U.S. Bureau of Labor Statistics, though the service-flows of "better" varieties persist longer. Estimated service-life densities miss too many low-value vehicles to be treated as survivor densities, so the Hulten-Wykoff regression survivor correction is insufficient. Averaging over age and date as conditioning variables yields reasonable unconditional life distributions that cluster close to a 13-year mean.

A tentative National Accounts application is offered, with numerical details of implementation consigned to a spreadsheet that is available upon request.

JEL Codes: C49, C65, D24, L62, M41

Keywords: used-car auctions, depreciation, obsolescence, service-lives, cross-entropy

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*Thanks to Barbara Fraumeni, Arnold Katz, Dylan Rassier, Marshall Reinsdorf, Jon Samuels, and Rachel Soloveichik for helpful conversations. All remaining errors and the views expressed here are mine, not necessarily those of the Bureau of Economic Analysis or the Department of Commerce. Please address correspondence to: Brian Sliker, U.S. Department of Commerce, Bureau of Economic Analysis, Washington, DC 20233. Email: brian.sliker@bea.gov Tel: (301) 278-9649

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This paper reports results of an analysis of Ford Windstar passenger vans sold at U.S. wholesale used-vehicle auctions. It models individual-level depreciation and shows how individuals fit into cohorts, it describes the sourcing and preparation of the data, and it details a novel adaptation of the underlying statistical procedure. Finally, it sketches how its findings may be applied to the National Accounts. Key results are:

1. For model-years '95-'06 — i.e., every model-year but one ever built of Ford Windstar and Freestar and Mercury Monterey passenger vans — "as-if-new" prices (part of the measurement of obsolescence) fell at an average annual trend rate of 10.3 percent. This rate was not constant, but sped up from 4.4 percent per year in the third quarter of 1994, when the available data begin, to 13.4 percent per year in the second quarter of 2007, when the data end. In simple dollar terms, a new '95 Windstar wholesaling for $18,000 in 1994:3 would have sold for $3,116 in 2007:2 had it been stored in pristine condition and never driven.

2. For model-years '95-'02, the focus of this report, the overall average "Trapped Geometric efficiency parameter" value that performs best, a=1.5, implies an individual-level age-efficiency profile that is mildly concave downward. The profile's shape is strikingly similar to the hyperbolic age-efficiency profile that the U.S. Bureau of Labor Statistics (BLS) has long used to estimate the efficiency of all used equipment. Efficiency parameters *restricted to particular model-years or varieties* vary from the overall average, with "nicer" varieties tending toward higher values of a (i.e., more concave efficiency profiles, implying a longer fraction of the service life where a van "runs well").

3. For calendar-year quarters 1997:1-2007:2, the best choice of the "own rate of return," r, varies among the choices .12, .15, and .18 without a clear pattern until 2000:4, after which r = .12 is always preferred. This may implicate too few choices, too widely spaced, in the grid-search for r. On the other hand, nominal interest rates of various maturities trended generally downward after 2000:1, so it's conceivable that declining finance rates and worsening as-if-new prices may have approximately offset each other.

4. As one might expect, there is a tradeoff between miles driven and service-lives, but the tradeoff's severity diminishes with age. Driving a three-year old van another thousand miles is associated with an eight-week reduction in its expected service-life, but driving an eight-year old van another thousand miles costs only two weeks of expected service. (However, three-year old vans sold at auction, revisited five years later, are not necessarily equivalent to eight-year old vans sold at auction at the time of the revisiting. See the next finding.)

5. The statistical procedure estimates its own conditional service-life densities for quarter-year - wide age-brackets at each calendar-year quarter. This overcomes a limitation of standard age-price regressions, which must import survival distributions to compensate for the censoring of zero-value

retirees (Hulten and Wykoff, 1981a). Yet the estimated densities do not exhibit the left-truncation expected of survival distributions, for there is next-to-no mass of auctioned vans with service-lives less than five years past their ages, at nearly all available ages. Estimates of *overall* service-life densities for each Windstar vintage/variety are found by averaging across the conditional densities, using as weights the auction transaction-counts of each narrow age $\times$ date bracket. Estimated densities for most of the twenty-three vintages/varieties treated here resemble Extreme Value densities. Every variety's mean is within 1.5 years of a composite all-Windstar mean service-life of 12.8 years, and every variety's standard deviation is within 1.4 years of a composite service-life standard deviation of 2.8 years.

The paper has six sections. A brief introduction sets out the main questions of this research and why the answers matter. It cites the data, which a few other authors have used, and the estimation method, which even fewer have used. It emphasizes depreciation as a plural process, not a single number. Then the first content section lays out a framework for measuring capital consistently across the individual and cohort levels. Along the way it talks through the technical terms, many of them mentioned already in the key findings. The second section briefly describes the dataset and how it is used, plus a few other papers that have used it or something similar. The third section details the statistical estimation methods, which are two: one a regression approach that leads to fitted values that have the interpretation of as-if-new prices, the other a modification of cross-entropy estimation to make it comport better with Hicksian aggregation. The fourth section discusses the estimation results and shortcomings. The conclusion recounts the main results, which call into question several longstanding conventions of capital measurement. To spare readers too many details, the first, second, and third sections all have appendixes. A final, long appendix that reads like a user-guide, plus a supplemental *Excel* file available upon request, work out the treatment of Ford Windstars in the National Accounts as if they were a separate asset, but readers need not limit their imaginations to beat-up old vans.

0. Introduction: Why Measure Depreciation and What to Measure

Depreciation is the loss of value experienced by capital assets as they age through time. At the macro level, it is the difference between national accounting agencies' headline number, the Gross Domestic Product, and the closest measurement we have of aggregate value-added in a given production period, *Net* Domestic Product. At the industrial level, it is the difference between gross investment, which includes a sizeable replacement component, and net investment, which is thought to transmit improved technologies. At the micro level, it represents funds a business must set aside to enable its continued operations into the future.

Depreciation may be caused by natural processes of decay, the wearing out of parts by (over)use, or maintenance (mal)practice — all of which affect originally-identical asset-type units differentially — or by changes in the broader environment that make an entire asset class or variety less useful or attractive or more costly to operate, regardless of how well individuals have been treated or whether they had even been removed from their packaging. The first, individual-focused depreciation is commonly called wear-and-tear or just "ordinary" depreciation; the second, broad-based sort is termed obsolescence. A prominent, but not exclusive, example of obsolescence is the devaluation of a serviceable, nearly-new generation of assets when a newer, better-featured variety of the same class is introduced. Measured wear-and-tear may shed light on the intensive-margin utilization of capital. Measured obsolescence offers a rear-window view of technological progress. We lose a lot by reducing these different channels to a single number, despite the savings — e.g., fewer data demands, simpler accounting, ease of exposition — obtainable from the standard Geometric model. When a flexible, internally-consistent micro-level modeling framework and plentiful, detailed data are both available, we should seize the chance to find out all we can about the processes in play.

This paper makes use of almost a million wholesale used-vehicle auction transactions of a narrow asset type — over twenty varieties of Ford Windstar and Freestar and Mercury Monterey vans — from nearly all the years when the vans were new. It infers the vans' ages (in principle, to the nearest day), and it backs out what they would have sold for had they been offered for sale as "new" *years after they actually were*. This "as-if-new price" construct is central to the analysis. The difference between the observed price of a genuinely new ("frontier") variety and the as-if-new price of an older variety is a clean (if narrow) gauge of obsolescence, while the difference between a variety's as-if-new price and the actual used-price of one of the variety's members measures the penalties of wear-and-tear. The paper constrains varieties' as-if-new prices, once established, to move in strict proportion, the better to treat all Windstar-type vans as a Hicksian aggregate, which has the practical benefit that the expected proportionate revaluation term is the same across all individuals' user-costs at any given date.

On a mature subset of the data, the paper then estimates the parameters that summarize individual wear-and-tear: an efficiency parameter that indirectly captures what fraction of the service-life delivers a "nice ride," an own rate-of-return (a finance rate, less the expected proportional revaluation), and the distribution of service-lives for vans of a certain age at a given date. Such narrow-window life-distributions can be combined into an unconditional distribution, so that the importation of external distributions to adjust age-price regressions for retirements, can be dispensed with. Estimation is by cross-entropy matching of many narrow-domain moments, but I have modified the technique to allow two independent processes (obsolescence and aging) to inform the same moments yet stay independent.

The detailed data and two-part estimation enable a range of findings, as summarized above. The consistency of the depreciation model at individual and cohort levels, and across the efficiency and price domains, enables consistent and meaningful treatment of Windstars as both productive and wealth capital in the National Accounts.

1. Age-Efficiency and Resale-Price Profiles: Trapped Geometric Individual Forms within Cohorts[1]

The Trapped Geometric model age-efficiency profile for an individual van drawn from a cohort / vintage / model-year (used interchangeably) is:

$$\phi(s,L) = \frac{e^{a\,s/L} - e^{a}}{1 - e^{a}} \qquad (s \le L, \text{ else } \phi = 0). \tag{1.1}$$

ϕ compares the service-flows (equivalently, the constant-price rental income) immediately accessible to an s-year-old individual asset destined to remain in service L years from its manufacture or installation, to the flows immediately accessible to the same individual when new. As s increases from 0 to L, ϕ decreases from 1 to 0. Depending on the value of the "efficiency parameter" a, the decrease might be negligible until the end (i.e., for $a \to \infty$), which is the "one hoss shay" productivity pattern, or (less extreme) slower while new than while old ($a > 0$), or equal at every age ($a \to 0$) for "straightline efficiency loss," or faster new than old ($a < 0$), or immediate ($a \to -\infty$), signifying "not an asset." This single-parameter versatility makes the form comparable to the individual-level hyperbolic age-efficiency profile used by the BLS, the U.S. Agriculture Department's Economic Research Service (ERS), the Australian Bureau of Statistics (ABS), and the Bank of Korea (BoK). Unlike the hyperbolic profile, the trapped-efficiency form has a dual resale-price profile, $\theta(s,L)$, which is easy to derive under constant-rate discounting:

$$\theta(s,L) = \frac{\int_s^L e^{-r(u-s)}\,\frac{e^{a\,u/L} - e^{a}}{1-e^{a}}\,du}{\int_0^L e^{-r(u-0)}\,\frac{e^{a\,u/L} - e^{a}}{1-e^{a}}\,du} = \frac{\frac{a\left(1-e^{r(s-L)}\right) - rL\left(1-e^{a(s/L-1)}\right)}{r(a-rL)(1-e^{-a})}}{\frac{a\left(1-e^{-rL}\right) - rL(1-e^{-a})}{r(a-rL)(1-e^{-a})}} = \frac{a\left(1-e^{r(s-L)}\right) - rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL}) - rL(1-e^{-a})} \qquad (s \le L, \text{ else } \theta = 0) \tag{1.2}$$

where r is the *own rate of return* — i.e., an appropriate finance rate, less the expected log growth-rate of the price the individual would fetch if it were somehow kept new.[2] Under progressive obsolescence, where new features introduced each model year push down such so-called "as-if-new" prices of older model-years, that log growth-rate could be quite negative, implying high r, possibly above 10 percent, even if finance rates are near zero. Like the age-efficiency profile, the resale-price profile also decreases from 1 to 0 as s increases from 0 to L, but the resale-price profile's value never exceeds the efficiency profile's value at the same age. Multiplying $\theta(s,L)$ by the as-if-new price for vintage or "variety" v at date t, P^0_{vt}, would approximate the actual date-t price of an s-year old asset from vintage/variety v. Multiplying instead by P^0_{tt}, the observed new price of the latest vintage (i.e., v=t), would overstate the used price, because obsolescence has caused $P^0_{vt} < P^0_{tt}$ for $t > v$.

This discussion has separated obsolescence, which compares new and as-if-new prices at the same date, from "ordinary" depreciation, which compares an individual at different ages within its own service-life. Because individuals age at different rates (owing to different L-values), ordinary depreciation is best approached at the level of individuals within a vintage/variety. On the other hand, $\theta(0,L) = 1$ irrespective of the value of L, so rolling individuals' s-values back to 0 would impose the Law of One Price across all as-if-renovated members of the same vintage/variety, implying obsolescence is best studied by comparing across new and as-if-new vintages

[1] So named by "trapping" an ordinary geometric profile (that is, $e^{(a/L)s}$, with $-a$>0 as the declining-balance rate, so that $-a/L$ is the continuous-time depreciation/deterioration rate) to zero when its age, s, reaches the service-life L: $e^{-(a/L)s} - e^{-(a/L)L}$, then normalizing by the value of the same difference when new (i.e., s=0): $(e^{as/L} - e^{a})/(1 - e^{a})$. The trapping procedure frees a to take any real value, though limits are necessary as a approaches ∞, $-\infty$, or 0.

[2] Expression (1.2) has limiting forms as a and r approach ∞, $-\infty$, and 0 singly or in combination, and as L approaches a/r.

rather than across individuals. Hall (1968) pointed out that age-effects, date-effects, and vintage-effects were not all identified in a log-linear age-price depreciation regression with annual data and went so far as to suggest economists not bother to distinguish obsolescence from ordinary depreciation. Having high-frequency data at both the individual and cohort levels of aggregation offers a way around Hall's concerns. We return to the matter below, following (1.5).

Transforming the individual resale-price profile to the corresponding rental-price profile, $\rho(s,L)$, is straightforward:

$$\rho(s,L) = r\frac{a\left(1-e^{r(s-L)}\right)-rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL})-rL(1-e^{-a})} - \partial\frac{a\left(1-e^{r(s-L)}\right)-rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL})-rL(1-e^{-a})}/\partial s = \frac{r(a-rL)\,(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}\frac{e^{a\,s/L}-e^{a}}{1-e^{a}} \quad (1.3)$$

Like the other two profiles, the rental-price profile declines to 0 as s increases from 0 to L. As with the discussion above regarding the resale-price profile, the product $P^0_{vt} \times \rho(s,L)$ should approximate an individual used-asset's actual rent. However, $\rho(0,L)$ — that is, the gray block in (1.3), equivalently the reciprocal of the denominator integral in (1.2) — is essentially a reciprocal function of L. So the individual-level user-cost — i.e., $P^0_{vt} \times \rho(0,L)$ — is larger, owing to a higher value of the new-asset depreciation rate, $\frac{r(a-rL)\,(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})} - r \geq 0$, for short-lived individuals than for long-lived ones. This is to be expected, as short-lived assets must earn greater rents in the little time they have to repay their purchase price. Note that $\int_s^L e^{-r(u-s)}\rho(u,L)\,du = \theta(s,L)$, as per (1.2), and that $\rho(s,L)/\rho(0,L) = \phi(s,L)$. At the level of an individual asset, then, each of forms (1.1), (1.2), or (1.3) implies the other two, so the system is sound. But it remains to estimate P^0_{vt}, which is not observed.

The statistician cannot realistically know an individual's L, but for productivity and national accounts purposes, the behavior of price and efficiency profiles at the *cohort* level built from individuals is enough.[3,4] Aggregating individual-level resale-price profiles up to the cohort level is conceptually easy:

$$\boldsymbol{\Theta}(s) = \underbrace{\int_0^s f(L)\times 0\;dL}_{zero!} + \int_s^\infty f(L)\;\theta(s,L)dL = \int_s^\infty f(L)\frac{a\left(1-e^{r(s-L)}\right)-rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL})-rL(1-e^{-a})}\,dL \quad (1.4)$$

though an algebraic expression for $f(L)$, the service-life density, might not be explicit. The cohort profile differs from the observed average resale-price profile for survivors only:

$$\widetilde{\boldsymbol{\Theta}}(s) = \int_s^\infty f(L|L>s)\theta(s,L)dL = \frac{\int_s^\infty f(L)\theta(s,L)\,dL}{\int_s^\infty f(L)\,dL} \quad (1.5)$$

[3] On the other hand, the case can be made that an individual's L can become *better known* by comparing the individual's performance vis-à-vis its neighbors over time (Sliker, 2015). An implication of improving knowledge of individual L is that *early* knowledge may be poor, so estimates of individual values may overconcentrate near the survivors' mean, reducing the informativeness of higher moments until individuals are better distinguished.

[4] In fact, many statistical agencies, BEA among them, do not track cohorts separately, but only the net stock, which sums across cohorts. Constant-rate (i.e., geometric) cohort-level depreciation is a sufficient but unnecessary condition for geometric stock-level depreciation. Diewert and Wei (2017) show that a stationary cohort-level efficiency profile, constant own rate of return, and constant growth-rate of investment are also jointly sufficient for geometric stock-level depreciation.

so the Hulten-Wykoff (1981a) correction to age-price regressions, whereby the relative-to-new resale-price ratios, which constitute the observations to be explained, are multiplied by the survivor function: $\Theta(s) = \widetilde{\Theta}(s)\times\int_s^\infty f(L)dL$, is justified.[5]

The discussion of obsolescence following (1.2) paused to allow development of individual- and cohort-level measures of capital that decay at different rates with respect strictly to age. This was permissible because the Law of One Price for as-if-new individuals boiled down realized obsolescence to a simple multiplicative factor, dependent on vintage/variety and calendar-time but equally applicable to all a cohort's members at any given date. But some price variation across apparently identical new individuals (and so, across as-if-new individuals) is inevitable, so to allow *some* play, replace a strict Law of One Price by statistical independence between as-if-new prices embedded in one distribution, and individual-level resale-price profiles embedded in another. (Recall the expected log-growth rate of as-if-new prices is already built into *r*.) Thus the expected resale price of an *s*-year old asset from cohort *v* as of date *t* may be written:

$$E\left(P_{v,t}^{s}\right) = \int_{\underline{P_{v,t}^{o}}}^{\overline{P_{v,t}^{o}}} g\left(P_{v,t}^{o}\right) P_{v,t}^{o}\, dP_{v,t}^{o} \times \int_{s}^{\infty}\int_{\underline{m}}^{\overline{m}} f(L,m)\,\frac{a\left(1-e^{r(s-L)}\right)-rL\left(1-e^{a(s/L-1)}\right)}{a\left(1-e^{-rL}\right)-rL\left(1-e^{-a}\right)}\,dm\,dL \qquad (1.6)$$

where the first term on the right is the average as-if-new price of a cohort-*v* asset as of date *t*, with density $g\left(P_{v,t}^{o}\right)$ running between reasonable bounds $\underline{P_{v,t}^{o}}$ and $\overline{P_{v,t}^{o}}$.[6] Treating $g\left(P_{v,t}^{o}\right)$ and $f(L,m)$ as independent is intentional: the ratio of as-if-new to genuinely-new prices remains a clean measure of obsolescence that way, even as a ratio of averages.[7] Moreover, the as-if-new price averages aren't buffeted by variations in *L*, so one may treat the cohorts in the net stock (of all Windstar, Freestar, and Monterey passenger vans) as forming a proper aggregate.[8]

Several estimating equations in this paper's statistical-technique section, farther below, have the flavor of (1.6). However, they sample only from transacted individuals, so their left-hand side moments might be more appropriately written as $E\widetilde{\left(P_{v,t}^{s}\right)}$, akin to (1.5). Likewise, their (marginal) $f(L)$ densities are really about survivors, i.e., $f(L|L>s)$, or even just the subpopulations of vans worth hauling to auction at various ages, $f(L|s)$.

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[5] The distributions of individuals resold at various ages might differ more generally than allowed by differences in the left-truncation severity of otherwise-identical survival densities. If so, the observed average resale-price profile is $\widetilde{\Theta}(s) = \int_s^\infty f(L|s)\theta(s,L)dL$, not $\int_s^\infty f(L|L>s)\theta(s,L)dL$, so Hulten and Wykoff's convenient correction would not go far enough. Instead, use conditional probabilities to construct $f(L) = \int_0^\infty f(L|s)f(s)dL$, where $f(s)$ is the unconditional density of ages. With $f(L)$ in hand, *then* reconstruct the cohort. The results of section four, below, imply conditional, not survival, densities.

[6] $\int_{\underline{P_{v,t}^{o}}}^{\overline{P_{v,t}^{o}}} g\left(P_{v,t}^{o}\right) P_{v,t}^{o}\, dP_{v,t}^{o}$ would also multiply ρ(s), the net-of-obsolescence cohort rental-price profile. *Cf.* Appendix A, below.

[7] By introducing $f(L,m)$, the joint density of service-lives and miles driven, where miles range between appropriate bounds $\underline{m}$ and $\overline{m}$, I have gone beyond the text, though the extension is reasonable. In equation (1.6), the joint density finds no function of miles to operate on, so the double integral marginalizes *m* out of $f(L,m)$, leaving $f(L)$.

[8] The "sport" option of a car, luring risk-loving drivers into more frequent accidents, is a counterexample.

2. Brief Description of the Data and Their Uses

The study's 373,496 transactions of used Ford Windstars were drawn from the "Raw Auction Data Set," a joint project of the National Automobile Dealers' Association (NADA) and the National Auto Auction Association (NAAA), containing records of 78,906,355 completed wholesale auction trades of automobiles, light, medium, and heavy trucks, and some trailers in the United States, Canada, and the Caribbean from January 2, 1991 through May 31, 2007.[9] Each transaction record includes the make (e.g., "Ford Truck"), model-year ("98"), submake ("Windstar"), series ("Windstar"), body-type or trim ("WGN 3.0L/LTD/LX/GL"), date of sale, region of sale, sale price, odometer reading, sale type, and first 11 digits of the 17-digit Vehicle Identification Number (VIN).[10] The final 6 digits are masked, which was a steep hurdle to inferring individual vehicle build-dates and (subtracting from the sales-dates) precise ages. The "Data Work" appendix details the steps involved to estimate individual build-dates and ages, as well as procedures to clean and organize the data.

A few others have used wholesale vehicle auction transactions, whether from NADA/NAAA or some of the major auction houses. Lacetera, Pope, and Sydnor (2012) note resale-price drops of \$100-\$200 whenever a car's odometer crosses a thousand- or ten-thousand -mile mark, indicating an "inattention bias" by retail customers that dealers factor into their own bids at wholesale auctions.[11] On the other hand, Sallee, West, and Fan (2016) find that *comparative* changes in the prices of otherwise-identical vehicles with different odometer readings (and thus different expected remaining service-years) in response to a change in the price of gasoline (which is assumed persistent) offset the capitalized incremental fuel costs one-for-one, implying buyers would respond optimally in the face of Pigouvian gas taxes. Larsen (2014) used private auction-house data to study the efficiency of subsequent back-and-forth bargaining over used vehicles that had *not* sold successfully at auctions.

The present paper only scratches the available wholesale auction data from NADA/NAAA. First, the vehicular focus is narrow: Ford Windstars only (renamed Freestars in 2004 and supplemented by the high-end Mercury Monterey). This limits the sources of heterogeneity to the distributions of as-if-new prices of Windstar model-years and trim levels, which will be constrained to move in lockstep to maintain the useful fiction of an aggregate, and to the within-trim distributions of individual service-lives.[12] (It also let me develop estimation procedures in a manageable setting.) Second, the price-and-miles moments to be matched to analytical expressions are also narrow, covering only what's available in 3-month age brackets $\times$ 3-month calendar-time brackets called "cells." Estimation takes place as a series of snapshots of nearly 1700 cells. Then depreciation follows from reconstructing (1.6), using the best-fit common values of a and r and numerical substitutions for cell-specific $f(L,m)$ and for a moving array of mean as-if-new prices. I did not account for heterogeneity that

[9] According to NADA's website, the data cover "80 percent of the nation's auction transactions every day from Manheim, ADESA, ServNet, ABC and key independent auction houses." The data set continues after May 2007, but BEA's purchase only extends that far.

[10] Of the seven auction types, manufacturers' auctions to their captive dealers and fleet auctions are both numerous but early in any model year's career; dealer auctions are numerous and represent typical sales where vehicles are well individuated; as-is, scrap, repossession, and unknown auction types are rare.

[11] Those discrete price-drops might frustrate the interpretation of a depreciation rate as fit by a typical log-linear age-price regression. I am hoping my procedure of matching short-interval sampling moments of prices and miles to analytical expressions, as described below, will "average around" the price discontinuities.

[12] Forcing mean as-if-new prices of all varieties of light motor vehicles to move together, and their distributional spreads also, is a much bigger ask, but it follows from treating light motor vehicles as an aggregate.

might arise from different auction types, as gasoline makes a vehicle fungible across auctions, nor did I treat auction regions differently. However, the model's informational requirements — i.e., that auction participants can form fairly tight estimates of individual service-lives — rule out the use of transactions of vans that are early in their careers (where manufacturer and fleet auctions dominate), which are too young to tell apart. This made the lion's share of included transactions dealer-to-dealer trades. See Appendix B for the details in boiling down the data to 373,496 useable transactions.

Data used for a single cell's estimation included: the price, age, transaction date, and miles driven of each included vehicle.

3. Statistical Estimation

Holding $g\left(P_{v,t}^{o}\right)$ independent from $f(L,m)$ has the practical advantage of allowing vintages' and varieties' mean as-if-new prices through time to be estimated separately from the more complicated treatment of age, miles, and survival that pertains to wear-and-tear. So this section is divided into two parts. The first part estimates mean as-if-new prices in a pseudo-poisson regression framework, while the second modifies the cross-entropy probability estimation method to handle independent distributions — i.e., $g\left(P_{v,t}^{o}\right)$ *versus* $f(L,m)$ — that participate in the same moments. Extra details of the both parts are consigned to Appendix C.

3.1 Pseudo-Poisson Regression of Prices

There was a time when an "age-price regression" meant the OLS projection of used assets' logged prices on the assets' ages, transaction dates, and their non-age quality-characteristics (skirting vintage as a date). The use of non-age characteristics (as opposed to the vintage year, which is collinear with age and date, in annual data) and time were intended to remove confounding variables from a clean (i.e., age-based) measure of depreciation, though Hall (1971) used the technique to infer the rate of quality improvement, shorn of age and time confounders. Without adjusting for retirements, which are a function of age, the simple loglinear regression does not measure fast enough depreciation. But Hall's approach to quality still makes sense, if we use functions of age (and miles) as statistical sponges to control for depreciation *and* retirements semi-parametrically, deferring their separation until later. Moreover, quality is embodied in vintages, so it is useful to track vintage dummies themselves through time. So consider the crowded but essentially familiar regression:

$$P_{v,t,i} = \mathbf{Exp}\Big[c + \beta_s s + \beta_m m + \beta_{ss} s^2 + \beta_{sm} s\times m + \beta_{mm} m^2 + \beta_{sss} s^3 + \beta_{ssm} s^2\times m + \beta_{smm} s\times m^2 + \beta_{mmm} m^3 + \sum_v \beta_v I_v + \sum_v \beta_{vs} I_v\times s + \sum_v \beta_{vm} I_v\times m + \sum_t \beta_t I_t + \sum_t \beta_{ts} I_t\times s + \sum_t \beta_{tm} I_t\times m\Big] + \varepsilon_{v,t,i} \quad (3.1.1)$$

where $P_{v,t,i}$ the price (in \$thousands) of individual *i* from vintage/variety *v* during date *t* (to the nearest calendar-year quarter), depends on the individual's age *s* (presumably to the nearest day) and odometer reading *m* (in thousands of miles). The regression's 44 included varieties and 51 included calendar-year quarters are each interacted with age and miles, which also enter separately in a saturated third-order fit.[13] The two sets of dummy variables probably induce residual clustering, so I clustered standard errors by the unions of both dimensions. Variety *v'*s mean as-if-new price at date *t* is:

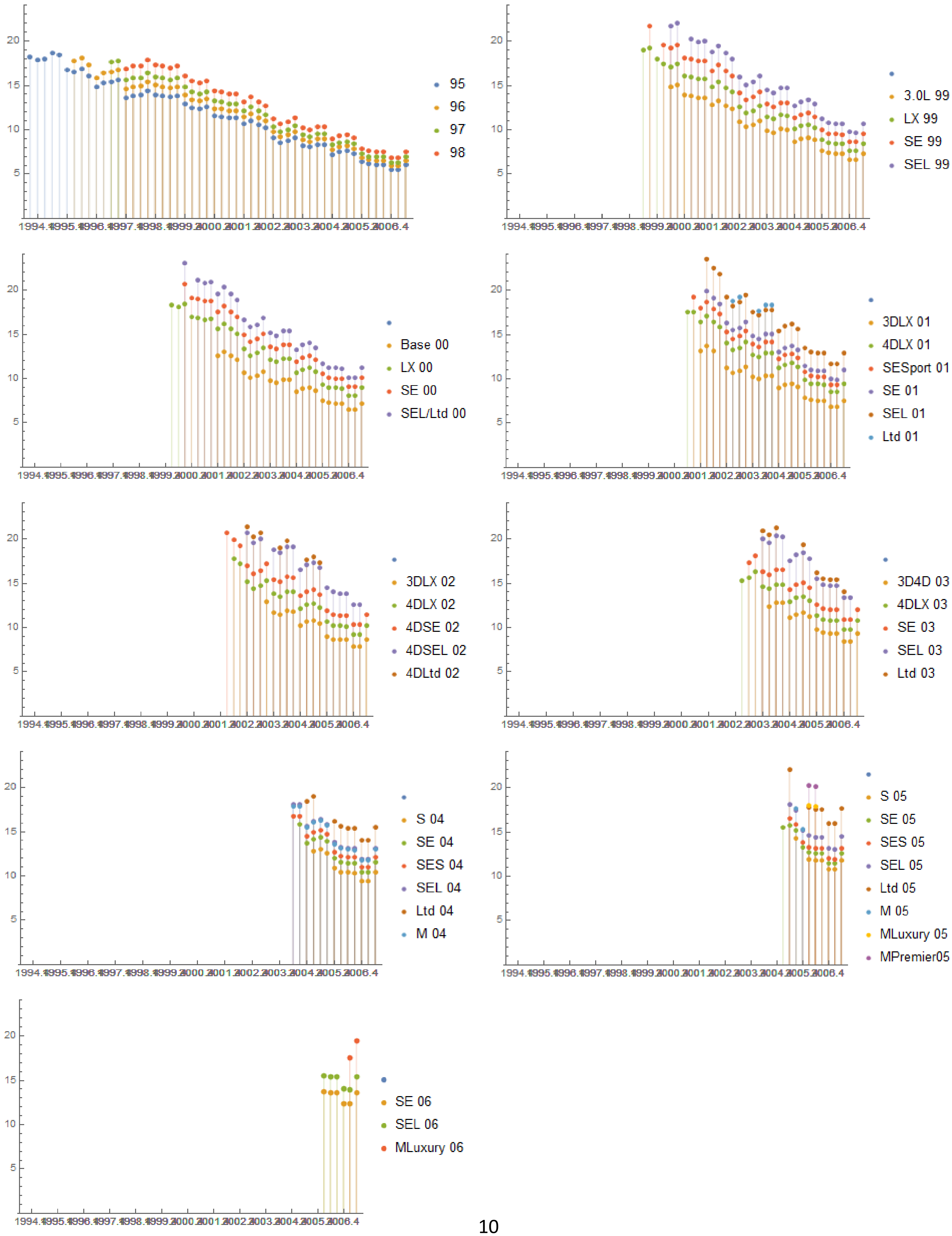
$$E(P_{v,t}^{o}) = \mathrm{Exp}(c + \beta_v + \beta_t) \quad \ldots^{14} \quad (3.1.2)$$

The exponential form implies that all varieties face the same basic inflation rate at date *t*, though the rate may vary across dates. Moreover, each variety's place in the quality ranking is fixed by its β_v. Hence the expected revaluation rate, which is subtracted from the finance rate to form an estimate of the own rate of return, is the same across all individuals. The following nine plots of 45 Windstar varieties' as-if-new prices through calendar-time demonstrate pronounced and persistent declines in as-if-new prices, averaging 10.3 percent per year, jostled by the quarters of the annual sales cycle:

[13] The '06 Mercury Monterey "Luxury" trim is the excluded variety; the excluded date is 2007:2. The unit of observation is transactions. I applied all 856,962 usable transactions, covering model-years '95-'06, including vans aged less than 3 years.

[14] Simple exponentiation, without log-normal or smearing-type transformations of the residuals, works here, for the pseudo-Poisson maximum likelihood (PPMLE) specification has $P_{v,t,i}$ rather than its log as the dependent variable (Santos Silva and Tenreyro, 2006).

Figure 1


95
96
97
98
3.0L 99
LX 99
SE 99
SEL 99
Base 00
LX 00
SE 00
SEL/Ltd 00
3DLX 01
4DLX 01
SESport 01
SE 01
SEL 01
Ltd 01
3DLX 02
4DLX 02
4DSE 02
4DSEL 02
4DLtd 02
3D4D 03
4DLX 03
SE 03
SEL 03
Ltd 03
S 04
SE 04
SES 04
SEL 04
Ltd 04
M 04
S 05
SE 05
SES 05
SEL 05
Ltd 05
M 05
MLuxury 05
MPremier05
SE 06
SEL 06
MLuxury 06

Notes on Fig. 1 (above). Each dot represents the evaluation of (3.1.2) for a given model-year ('95-'06) and variety (e.g., 3.0 Liter, Monterery Luxury, etc.) for the calendar-quarters the variety was present in the data (as early as 1994:III and as late as 2007:II) — i.e., cohort- and variety-specific as-if-new prices, in \$1000s. Horizontal-axis fourth-quarter marks show when new model-year sales typically commence.

The big regression's R^2 of 94 percent is adequate, and disposing of the saturated, non-dummied age and mileage terms leaves ample variation for *within-cell* variation to estimate aging and retirements. Each fitted $E(P^0_{v,t})$ will anchor subsequent estimation of its own cells' averaged age-price profiles. To approximate the *trend* rate of revaluation (which I'll take as the *expected* rate), I regressed the log of the fitted, left-hand side of (3.1.2) — i.e., $\hat{c} + \hat{\beta}_v + \hat{\beta}_t$ — against vintage/variety dummies and a quadratic time trend.[15] The relevant *point values*:

$$\hat{c} + \hat{\beta}_v + \hat{\beta}_t = \{\text{vintage/variety dummy variables}\} - \mathit{0.086437}\ (t\text{–}2000) - \mathit{0.00385589}\ (t\text{–}2000)^2 \qquad (3.1.3)$$

imply an expected rate of change:

$$\partial ln\mathrm{E}(P^o_{v,t})/\partial t = -\ 0.086437 - 0.00771177\ (t\text{–}2000) \qquad (3.1.4)$$

that annualizes to –4.4 percent in 1994:3, when the data were new and the Windstar's prospects bright, *versus* the catastrophic –13.4 percent by 2007:2, when the data are finished and the whole line had been cancelled.[16] Dropping the quadratic term from the trend regression yields a constant rate that annualizes to 10.31 percent. The trend rate of revaluation does not directly figure into the rest of the paper, but by removing it from a trust-worthy estimate of the own rate of return r, one may infer the operative finance rate.

The first part of the Appendix C begins by describing steps to form *distributions* of as-if-new prices, of which $\mathrm{Exp}(\hat{c} + \hat{\beta}_v + \hat{\beta}_t)$ are the means.

3.2 Modified Cross-Entropy Fits of Cell-Level Micro-Moments

To estimate the depreciation and survival of cohorts built up from individuals, it is necessary to uncover plausible statistical distributions from which the individuals were drawn. Following the reasoning developed at (1.6), above, the sought-after distributions are $g\left(P^o_{v,t}\right)$, the density of as-if-new prices of vintage/variety v at date t about their mean (3.1.2), and $f(L,m|s,t)$, the joint density of service-lives and miles driven of vehicles of vintage/variety v, from the narrow box of vans of ages $\underline{s} \le s \le \overline{s}$ resold at transaction-dates $\underline{t} \le t \le \overline{t}$ (both ranges ranges a quarter-year wide). In principal, $f(L,m|s,t)$ might be a survival density, i.e. $f(L,m|L>s,t)$. In the discrete-probability framework developed here, I'll represent $g\left(P^o_{v,t}\right)$ as lower-case p_n (n for "as-if-new") and $f(L,m|s,t)$ as $p_{l,m}$.

Likelihood-based statistical methods fit the parameters that particularize a member of some statistical family, such as a Weibull or LogNormal distribution, but they are less accessible when distributions are implicit. And while we might possess (and should find some way to use) prior intuitions about the shapes of the as-if-new and lives-and-miles densities, the paucity of recent findings on service-life distributions (Marston *et al.*, 1953), as

[15] The vintage/variety dummies correlate highly with $\hat{\beta}_v$, yet I did not net out vintage/variety dummies and $\hat{\beta}_v$ from both sides of the regression, since the variety and date dummies in (3.1.1) would not have been orthogonal in regression (3.1.1)'s unbalanced panel. I have not presented the OLS standard errors or t-ratios, which are too good to be believed.

[16] For 1994:III, I used 1994.625 which is midway between 1994.5 and 1994.75. Similarly, for 2007:II, I used 2007.375.

well as the statistical novelty of as-if-new price distributions, urge caution. Could the data structure the distributions, instead of the other way around?

Recent developments in information theory (Golan, Judge, and Miller, 1996) can help. In particular, the method of cross-entropy, which favors distributions that are *near* one's prior intuitions — provided the data's useful features are met — can generate empirically fruitful distributions and summarize the degree to which the data favor various test values of the key system-wide depreciation parameters *a* and *r*. In its primal form, the method manipulates sets of probabilities to minimize the expected proportional differences (i.e., the "Kullback-Leibler discrepancy") between the acted-on sets and corresponding sets of target probabilities (mis)named as priors, such that constraints involving the acted-on sets (typically moment equalities) are satisfied.[17] In its dual form, wherein the acted-on probabilities have been optimized out and replaced by exponential functions of the data domain and Lagrange multipliers, the solution mechanics typically involve globally concave maximizations with respect to the multipliers and are amenable to standard applications of the Newton-Raphson method.

"Priors" needn't be a dirty word. Where decent data are present to form accurate priors (e.g., several moments of odometer readings to construct marginal prior densities of miles), I use them. (Then proper posterior fits, given the same moments from which the priors were drawn, should confirm the priors exactly.) Where useful data are not available, I supply my own hunches or preferences, albeit somewhat flattened. In either case, the priors are specified as exponential functions that are algebraically compatible with the functional forms of the acted-on probabilities.[18] That way, the probabilities' final values, which include the digested priors, are numerically the same irrespective of the priors' contents. So *good* priors in cross-entropy estimation simply start the overall acted-on probabilities closer to where they will finish.

The first part of Appendix C continues the discussion of priors and describes how I selected them for the joint distribution of service-lives and miles.

A second practical reason for keeping $P^0_{v,t}$ independent from *L*-and-*m* is to reduce the number of system-wide support points in discrete cross-entropy estimation from the *product* of the cardinalities of $P^0_{v,t}$, *L*, and *m*, to the lesser *sum* of the cardinality of $P^0_{v,t}$ and the product of the cardinalities of *L* and *m*. This reasoning permits estimation of the probabilities of each age-quarter $\times$ date-quarter cell *separately*, given the already-fit $E(P^0_{v,t})$ — called $\hat{P}^o_{v,t}$ in (3.2.1), below, to save space — and *test values* of the efficiency parameter *a* and own rate-of-return *r*. This allows observing the development of joint *L*-and-*m* distributions across ages and dates, but at the stiff cost of repetitive estimates of thousands of age$\times$date cells.

Within every cell, I fit the joint distribution of unobserved service-lives and observed miles, together with the independent distribution of as-if-new prices, to match uncentered moments of prices and miles, by minimizing one of three cumulative levels of the constrained cross-entropy (per the cell's likely informativeness). The formal minimization program follows on the next page.

[17] A cottage industry has grown up to draw all the connections between Bayesian and entropy estimation. *Cf*. Giffon (2009). Operationally, Bayesian approaches combine maximum likelihood estimates with priors to form posteriors, which are then appropriate priors the next time fresh data are obtained, while minimum cross-entropy "tilts" what would have been maximum likelihood estimates "just enough" toward (nonuniform) priors.

[18] In Bayesian estimation, algebraically compatible priors are called "conjugate," but their use is quite different.

$$\min_{\{p_{l,m}\},\{p_n\}} \sum_l \sum_m p_{l,m} ln\left(\frac{p_{l,m}}{q_l q_m}\right) + \sum_n p_n ln\left(\frac{p_n}{q_n}\right) - \lambda^{o}_{lm}\left(\sum_l \sum_m p_{l,m} - 1\right) - \lambda^{o}_{n}\left(\sum_n p_n - 1\right) \tag{3.2.1}$$

$$-\lambda_{\mathrm{N}}\left(\sum_n p_n n - 1\right) - \lambda_{\mathrm{P1}}\left(\hat{P}^{o}_{\mathrm{v,t}}\left(\sum_n p_n n\right)\left(\sum_l \sum_m p_{l,m}\theta_l\right) - P1\right) - \lambda_{\mathrm{M1}}\left(\sum_l \sum_m p_{l,m} m - M1\right)$$

$$-\lambda_{\mathrm{P2}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{2}\left(\sum_n p_n n^2\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^2\right) - P2\right) - \lambda_{\mathrm{P1M1}}\left(\hat{P}^{o}_{\mathrm{v,t}}\left(\sum_n p_n n\right)\left(\sum_l \sum_m p_{l,m}\theta_l\, m\right) - P1M1\right) - \lambda_{\mathrm{M2}}\left(\sum_l \sum_m p_{l,m} m^2 - M2\right)$$

$$-\lambda_{\mathrm{P3}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{3}\left(\sum_n p_n n^3\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^3\right) - P3\right) - \lambda_{\mathrm{P2M1}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{2}\left(\sum_n p_n n^2\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^2\, m\right) - P2M1\right)$$

$$-\lambda_{\mathrm{P1M2}}\left(\hat{P}^{o}_{\mathrm{v,t}}\left(\sum_n p_n n\right)\left(\sum_l \sum_m p_{l,m}\theta_l\, m^2\right) - P1M2\right) - \lambda_{\mathrm{M3}}\left(\sum_l \sum_m p_{l,m} m^3 - M3\right)$$

$$-\lambda_{\mathrm{P4}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{4}\left(\sum_n p_n n^4\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^4\right) - P4\right) - \lambda_{\mathrm{P3M1}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{3}\left(\sum_n p_n n^3\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^3\, m\right) - P3M1\right)$$

$$-\lambda_{\mathrm{P2M2}}\left({\hat{P}^{o}_{\mathrm{v,t}}}^{2}\left(\sum_n p_n n^2\right)\left(\sum_l \sum_m p_{l,m}{\theta_l}^2\, m^2\right) - P2M2\right) - \lambda_{\mathrm{P1M3}}\left(\hat{P}^{o}_{\mathrm{v,t}}\left(\sum_n p_n n\right)\left(\sum_l \sum_m p_{l,m}\theta_l\, m^3\right) - P1M3\right) - \lambda_{\mathrm{M4}}\left(\sum_l \sum_m p_{l,m} m^4 - M4\right)$$

where:

$\{p_{l,m}\}$ is the joint distribution of service-lives (*l*) and miles (*m*) over a suitable bivariate grid, which the minimization attempts to bring "close" to the target/prior distribution $\{q_l\}\times\{q_m\}$;

$\{p_n\}$ is the univariate distribution of as-if-new prices (*n*) about their mean $\hat{P}^{o}_{\mathrm{v,t}}$ over a coarse grid, which the minimization attempts to bring "close" to the target/prior distribution $\{q_n\}$;

λ^{o}_{lm} and λ^{o}_{n} are Lagrange multipliers imposing the requirement that $\{p_{l,m}\}$ and $\{p_n\}$ are both proper;

λ_{N} is the Lagrange multiplier imposing the requirement that the range of a cell's as-if-new prices indeed average to the already-estimated mean as-if-new price $\hat{P}^{o}_{\mathrm{v,t}}$;

$\lambda_{\mathrm{P1}}, \lambda_{\mathrm{M1}}, \lambda_{\mathrm{P2}}, \lambda_{\mathrm{P1M1}}, \lambda_{\mathrm{M2}}, \lambda_{\mathrm{P3}}, \lambda_{\mathrm{P2M1}}, \lambda_{\mathrm{P1M2}}, \lambda_{\mathrm{M3}}, \lambda_{\mathrm{P4}}, \lambda_{\mathrm{P3M1}}, \lambda_{\mathrm{P2M2}}, \lambda_{\mathrm{P1M3}}, \lambda_{\mathrm{M4}}$ are Lagrange multipliers imposing the requirements that *modeled* uncentered mixed moments of resale prices and miles match their corresponding sample averages for the transactions enclosed within the cell, which are:

P1 = average transaction price
M1 = average odometer reading (miles)

P2 = average of *squared* transaction prices
P1M1 = average of products of transaction prices and miles
M2 = average of *squared* miles

P3 = average of *cubed* transaction prices
P2MI = average of the products of squared transaction prices by miles
P1M2 = average of the products of transaction prices by squared miles
M3 = average of *cubed* miles

P4 = average of transaction prices each taken to the fourth power
P3M1 = average of the products of cubed transaction prices by miles
P2M2 = average of the products of squared transaction prices by squared miles
P1M3 = average of the products of transaction prices by cubed miles
M4 = average of miles each taken to the fourth power, respectively; and

$\hat{P}^{o}_{v,t}$ is the estimated mean as-if-new price of vintage/variety v at date t
ℓ is a grid-value of the service-life random variable, from a cell's top age-boundary $\bar{s}$ to 25 years, by quarters
m is a grid-value of the miles-driven random variable, in 2000-mile increments
n is a grid-value of the as-if-new price random variable, from .78 $\hat{P}^{o}_{v,t}$ to 1.30$\hat{P}^{o}_{v,t}$ (1.22 $\hat{P}^{o}_{v,t}$) in 7 (6) increments

$\theta_l = \dfrac{a\left(1-e^{r(s^*-l)}\right)-rl\left(1-e^{a(s^*/l-1)}\right)}{a\left(1-e^{-rl}\right)-rl\left(1-e^{-a}\right)}$ is the individual-level resale-price (1.2) as a fraction of $\hat{P}^{o}_{v,t}$ (3.1.2).
(I set s^* to the average age of transactions in the cell.[19])

The estimation problem is less complicated than it looks. First, not every cell was forced to fit all 17 Lagrange multipliers included in the white, gray, and cyan zones of (3.2.1). Some cells matched only the 12 constraints in the white and gray zones (up to cubic terms); others only the 8 constraints in the white zone (up to squared terms).[20] Second, holding out a and r in θ_l as temporarily fixed parameters, rather than throwing them in with the λ's to be estimated all together, was a considered choice, because it allowed estimating each cell (out of potentially hundreds) separately, in a small-to-medium -sized problem; a joint procedure holding a fixed across all cells of a vintage/variety and r fixed across all cells at the same date is likely insuperable. The *best* values of a and r would minimize the observation-weighted sum of cross-entropies across all admissible cells, via a 2-way grid search. One might instead treat a and r as distributions themselves, but that is tricky: equal spacings of the support points of a or r translate into distinctly *unequal* implied prior probabilities of θ_l; further, r is multiplicatively bound up with the service life. Third, deciding which cells are admissible requires some care: populous early cells should be avoided if their implied lifespan densities have not settled down. Fourth, the independence of priors $\{q_l\}\times\{q_m\}$ for the joint distribution of lifespans and miles is implausible and sure to be rejected, but it is convenient and no worse than the implied bivariate uniform priors of maximum likelihood. Fifth, as stated earlier, forcing the distribution of as-if-new prices to stay independent of the joint distribution of unobserved service-lives and observed miles has the two advantages of keeping the user-cost's revaluation term the same for everyone and of conserving computer memory, as the total number of support points is held to the *sum* of each distribution's support points, not the *product*. (Things would be worse yet were as-if-new prices, lifespans, miles, a, and r all treated as jointly dependent distributions.)

However, keeping the as-if-new price distribution independent of the lives-and-miles distribution complicates cell-by-cell estimation. There is a standard formalism for bringing a *primal* cross-entropy constrained minimization problem such as (3.2.1) into the ambit of well-understood unconstrained optimization routines: First-order conditions with respect to individual probabilities have closed-form inverses akin to multinomial logit; the inverses express the probabilities in terms of the Lagrange multipliers and distributional support-points. Setting the products of the expressed probabilities and support-points equal to the sample moments gives a just-identified system of estimating equations with a friendly Hessian, readily solved by Newton-type methods, and the Hessian's inverse supplies the asymptotic standard errors. So far, so good.

---

[19] In principle, each cell's s has its own density, which I found could be modeled pretty well by $h(s) = \gamma\dfrac{e^{\gamma s}}{e^{\gamma\bar{s}}-e^{\gamma\underline{s}}}$, where γ is the density's single parameter and $\underline{s}<\bar{s}$ are the cell's age-boundaries, typically a quarter-year apart. But the complications of calculating *triple* weights, especially for higher-powered moments, were not worth the slight increases in accuracy over the simpler average s^*.

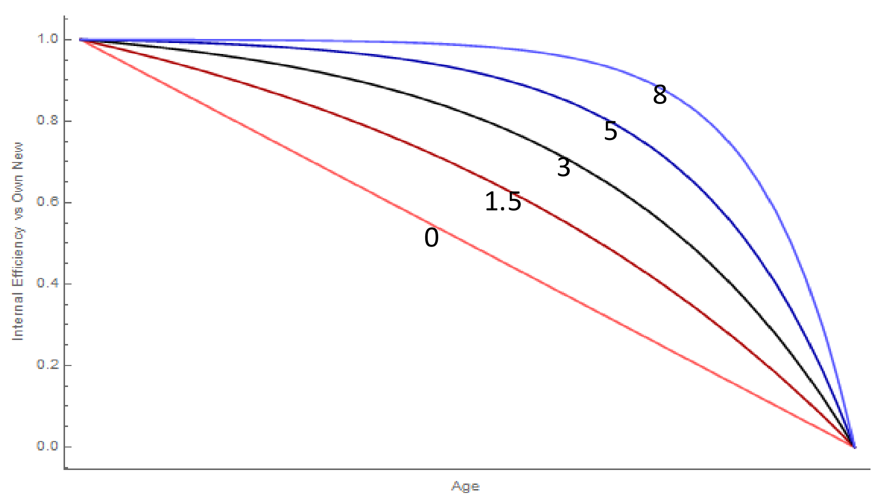
[20] *Cf.* Appendix C.1, below, in which the success of univariate maximum entropy fits of the miles distributions set the mold for the estimations carried out here.

The formalism still applies when two or more of the distributions are independent, but if they are implicated in some of the same moments (i.e., P1, P2, P1M1, P3, P2M1, P1M2, P4, P3M1, P2M2, and P1M3, but not M1, M2, M3, or M4, nor the constraint flagged by λ_N, guaranteeing that $\hat{P}^{o}_{v,t}$ really is the mean of $\{p_n\}$ for variety *v* at date *t*), then the resulting solved-out probabilities for one set of random variables will carry along "side-moments" that contain the unsolved probability-values of the *other,* independent set of random variables. As a consequence, the now-entangled estimating equations get doubled up: *twice* a moment equals the estimating equation in terms of the first distribution (e.g. lives-and-miles), which contains within it the side-moments of the second (as-if-new prices); *plus* the estimating equation in terms of the second distribution, seeded with the side-moments of the first. The Hessian is similarly complicated, though its eigenvalues remain friendly. Since each step of a Newton-type procedure updates the Lagrange multipliers of the parameterized probabilities, one must pause between steps to allow the unparameterized numerical values of the unsolved *other* distribution to "catch up" to the values of their parametrized counterparts, using a fixed-point algorithm.

The second part of the Appendix C describes in greater detail the steps to minimize (3.2.1) when the independent probabilities don't keep to themselves.

This version of the study did not iteratively optimize over efficiency parameter *a* and own rate-of-return *r* as with the Lagrange multipliers, not even within each cell. Instead, I selected a two-way grid of five *a*-values (i.e., 0, 1.5, 3, 5, and 8) implying roughly equally-spaced individual efficiency profiles, by three *r*-values (i.e, .12, .15, and .18) that I thought agreed with the finding of 10.31 percent average annual devaluation of as-if-new prices, following (3.1.4), above. The efficiency profiles hold for all service-lives:

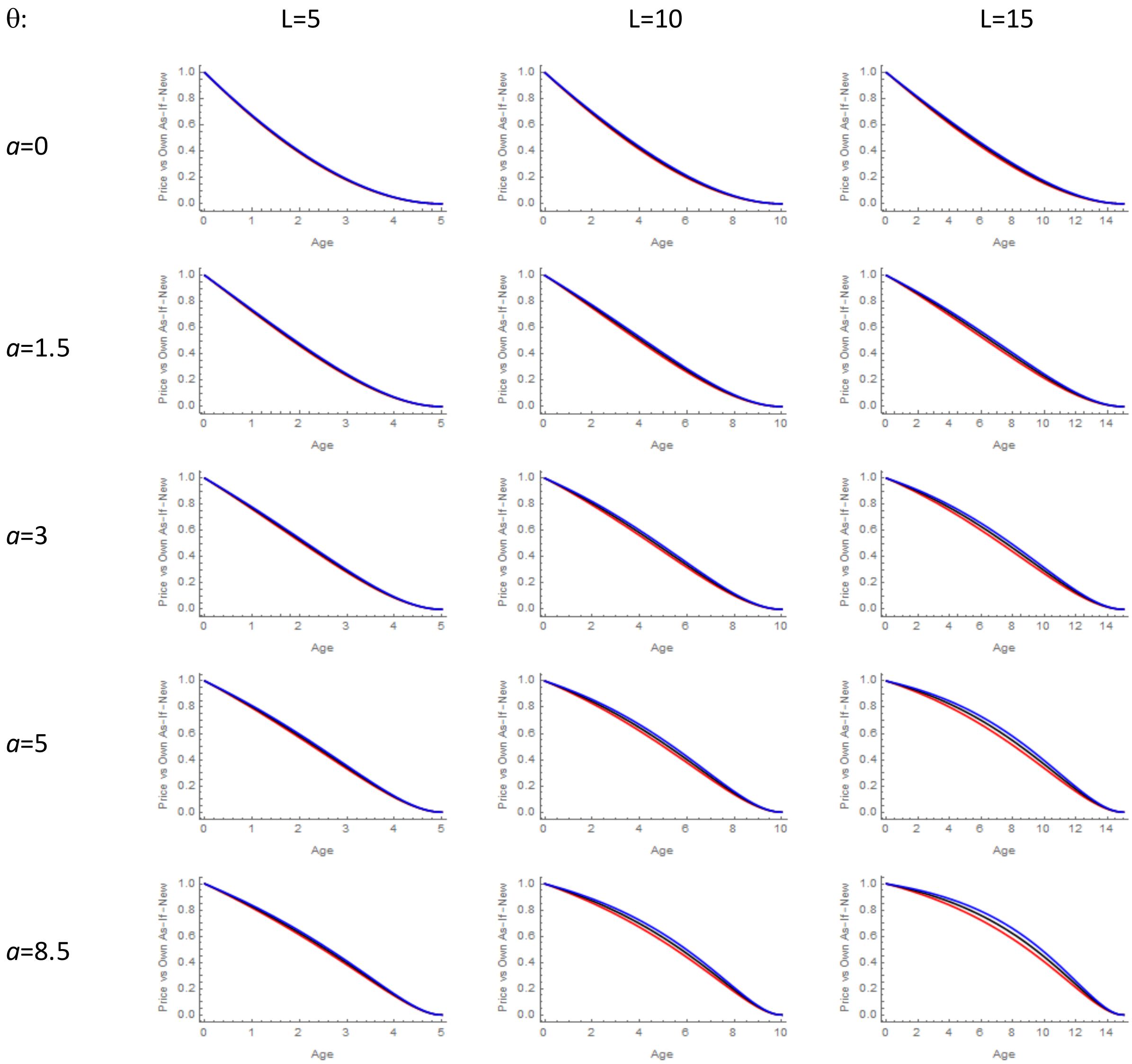
Figure 2



Notes on Fig. 2. Each schedule traces individual age-efficiency profile $\frac{e^{a\,s/L}-e^{a}}{1-e^{a}}$ *(1.1), which models the declining service-flows (and rental income) of an individual asset as it ages through time, relative to an identical version of itself that also moves through time but does not age. The next evenly spaced curve above "a=8" would be nearly one-hoss shay (for which a→∞).*

The own rates-of-return (i.e., .12, .15, and .18) would be considered widely spaced in financial discussions, but they had the effect of merely nudging individual resale-price profiles that were largely set by *a* and *L*. In fifteen plots of resale-price profiles just below, gaps between the profiles for *r* = .12 (red and always the lowest), *r* = .15 (black and in the middle), and *r* = .18 (blue and highest) only become visible for large values of *a* and *L*:

Figure 3



Notes on Fig. 3. Each schedule traces individual resale-price profile $\frac{a\left(1-e^{r(s-L)}\right)-rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL})-rL(1-e^{-a})}$ *(1.2), which models the declining price of an individual asset as it ages through time, relative to an identical version of itself that also moves through time but does not age. The value of a is important in shaping the profiles, the value of L less so, the value of r only slightly — to the point that it is hard to discern the three schedules present in each of the 15 plots. This suggests r will not be well identified in the statistical estimation.*

Searching the 5×3 grid over each of the qualifying cells in all 23 vintage/varieties of model-years '95-'02 led to 23,262 cross-entropy minimizations.[21] 21,699 of them converged, each with its own Lagrange-multiplier values and implied densities of as-if-new prices and of service-lives and miles driven.

[21] This is slightly less than 15 times the number of cells listed in Table B.1, as a few cells were dropped in the construction of cell-specific prior-densities for miles (*Cf.* Appendix C.1), and because I neglected (a,r) = (8.5, .15) for the '95 variety.

4. Results

The preferred specification is the joint choice of the efficiency parameter (*a*) and own rate of return (*r*) that minimizes the cross entropy between the resultant $p_{l,m}$ and p_n distributions and the original $q_l \times q_m$ and q_n target/priors. It is impossible to report each cell's preferred specification, and inevitable that the randomness of the data would obscure any sense of pattern across nearly 22,000 fits. Instead, the large Tables 1 and 2 on the next two pages show *averaged* cross-entropies with respect to *a*-values across all 23 vintage/varieties, and the same with respect to *r*-values across all 42 calendar-year quarters. The tables highlight the values of *a* or *r* that allowed the smallest Kullback-Leibler discrepancies (whether weighted by counts of cells or of transactions).

In Table 1, it is evident that early Windstars were not very good. (*Cf.* model-years '96-'98, where *a*=0 prevails.) But it is also apparent, once varieties are reported separately ('99 and later), that the "nicer" varieties, shown toward the right side of the page (the ordering follows the varieties' relative as-if-new prices), favor higher values of *a*. Colloquially, they deliver a "better ride" for a greater fraction of their service lives. Were one forced to pick a single *a* for *all* successfully converged '95-'02 Windstar cells, one would find:

Summary Table for *a* Values

'95-'02	# cells	# obs	cell-wts	obs-wts
0	4018	898957	0.883	0.872
1.5	4458	1003551	0.843	0.860
3	4522	1032956	0.931	0.957
5	4514	1019228	1.089	1.123
8.5	4187	920909	1.296	1.347

The overall preferred value, *a* = 1.5, is noteworthy, because the plot of the individual-level Trapped Geometric age-efficiency profile lines up almost exactly with the plot of BLS' individual-level Hyperbolic age-efficiency profile when the efficiency parameter for the latter, called β, is set to 0.5, which is the value BLS assigns to all equipment assets (including motor vehicles). However, BLS makes no provision for realized obsolescence, which as we have seen is not negligible for Windstars.

Table 2 is less appealing. The preferred own-rate of return bounces around until 2000:4, after which it is stuck at 12 *ln*-points (≈12.75 percent), the lower boundary of the grid space. Maybe a finer and wider grid would have allowed for smoother transitions and a better chance to fit the own rate well, though Figure 3, above, suggests skepticism. Nonetheless, an overall compulsory single rate works out as:

Summary Table for *r* Values

1997:1-2007:2	# cells	# obs	cell-wts	obs-wts
0.12	7272	1637610	0.960	0.978
0.15	7327	1654561	1.003	1.024
0.18	7100	1583430	1.060	1.090

On principle, I would resist imposing a constant *r* = .12. Not only are the estimates suspect, but the interest rates and new-vehicle prices one might use to infer *flexible* own rates of return are more readily available than a big once-and-done-purchase of auction data (Copeland, Dunn, and Hall, 2011). Appendix D, which attempts a national-accounts application, does accede to the constant *r*.

Table 1: Preferred *a*-Specifications for Each Vintage and Variety

95	# cells	# obs	cell-wts	obs-wts
0	661	180415	0.928	0.907
1.5	700	192478	0.782	0.804
3	702	192831	0.810	0.852
5	697	191197	0.921	0.976
8.5	462	126635	1.032	1.091

96	# cells	# obs	cell-wts	obs-wts
0	551	135127	0.818	0.818
1.5	567	141169	0.820	0.867
3	575	143541	0.913	0.967
5	580	146894	1.060	1.117
8.5	573	143952	1.256	1.355

97	# cells	# obs	cell-wts	obs-wts
0	173	25205	0.800	0.819
1.5	178	26489	0.817	0.853
3	177	26343	0.951	0.982
5	176	25996	1.131	1.165
8.5	173	25760	1.354	1.388

98	# cells	# obs	cell-wts	obs-wts
0	532	216002	0.718	0.753
1.5	543	223341	0.843	0.881
3	548	225647	1.022	1.058
5	548	224993	1.219	1.258
8.5	521	213119	1.416	1.460

30L99	# cells	# obs	cell-wts	obs-wts
0	84	6166	0.634	0.638
1.5	89	6481	0.710	0.719
3	91	7170	0.887	0.921
5	93	7065	1.100	1.133
8.5	91	6780	1.334	1.387

LX99	# cells	# obs	cell-wts	obs-wts
0	249	100397	0.793	0.898
1.5	251	99717	0.839	0.839
3	253	100870	1.019	1.008
5	249	93270	1.222	1.222
8.5	245	92758	1.424	1.433

SE99	# cells	# obs	cell-wts	obs-wts
0	235	26605	0.894	1.074
1.5	252	31647	0.847	0.849
3	252	31647	0.974	0.943
5	251	31364	1.156	1.126
8.5	251	30631	1.353	1.332

SEL99	# cells	# obs	cell-wts	obs-wts
0	75	3762	1.065	1.122
1.5	92	7249	0.943	0.993
3	92	7267	0.989	0.982
5	93	7305	1.142	1.124
8.5	90	7179	1.307	1.293

Base00	# cells	# obs	cell-wts	obs-wts
0	111	7420	0.778	0.857
1.5	110	7399	0.852	0.898
3	116	8346	1.072	1.125
5	109	7263	1.290	1.350
8.5	100	6296	1.523	1.626

LX00	# cells	# obs	cell-wts	obs-wts
0	256	78228	0.778	0.842
1.5	256	79654	0.808	0.809
3	267	95756	0.994	0.968
5	260	87941	1.209	1.187
8.5	259	82463	1.481	1.633

SE00	# cells	# obs	cell-wts	obs-wts
0	218	22036	0.988	1.115
1.5	242	34520	0.839	0.917
3	240	34742	0.920	0.884
5	242	34892	1.095	1.024
8.5	241	34733	1.338	1.221

SELLtd00	# cells	# obs	cell-wts	obs-wts
0	151	14258	1.135	1.168
1.5	210	30319	0.781	0.787
3	214	32171	0.741	0.655
5	216	32352	0.851	0.726
8.5	214	31476	1.009	0.868

3DLX01	# cells	# obs	cell-wts	obs-wts
0	123	10366	0.871	0.951
1.5	135	11621	0.786	0.842
3	133	11168	0.891	0.957
5	133	10973	1.083	1.154
8.5	127	10418	1.299	1.365

4DLX01	# cells	# obs	cell-wts	obs-wts
0	184	36240	0.825	0.886
1.5	190	41555	0.799	0.803
3	192	42575	0.967	0.954
5	195	44013	1.176	1.152
8.5	184	39478	1.411	1.354

SESport01	# cells	# obs	cell-wts	obs-wts
0	35	1714	1.529	1.597
1.5	63	5256	1.064	1.228
3	69	6506	0.898	0.961
5	71	6805	0.932	0.918
8.5	70	6796	1.039	1.001

SE01	# cells	# obs	cell-wts	obs-wts
0	108	8216	1.169	1.280
1.5	139	16058	0.856	0.953
3	139	16320	0.846	0.844
5	139	16523	0.979	0.952
8.5	141	16725	1.258	1.158

SEL01	# cells	# obs	cell-wts	obs-wts
0	41	2323	1.382	1.422
1.5	83	6667	1.071	1.159
3	84	6775	0.972	1.014
5	86	6841	1.078	1.105
8.5	85	6743	1.305	1.306

Ltd01	# cells	# obs	cell-wts	obs-wts
0	0	0	NA	NA
1.5	8	391	1.419	1.442
3	12	579	1.441	1.449
5	12	579	1.415	1.429
8.5	12	579	1.516	1.532

3DLX02	# cells	# obs	cell-wts	obs-wts
0	80	6829	1.192	1.279
1.5	89	7759	0.933	0.978
3	98	9080	0.952	0.990
5	99	9214	1.077	1.103
8.5	95	9195	1.319	1.330

4DLX02	# cells	# obs	cell-wts	obs-wts
0	100	14646	0.955	1.024
1.5	111	18952	0.858	0.918
3	106	17650	0.974	0.991
5	104	17153	1.177	1.182
8.5	92	12575	1.396	1.393

4DSE02	# cells	# obs	cell-wts	obs-wts
0	41	2472	1.216	1.231
1.5	91	10036	1.163	1.305
3	96	10806	0.984	1.033
5	97	11520	1.025	1.008
8.5	97	11520	1.154	1.106

4DSEL02	# cells	# obs	cell-wts	obs-wts
0	10	530	1.442	1.481
1.5	50	4256	1.233	1.220
3	54	4464	1.052	1.072
5	52	4373	1.102	1.157
8.5	52	4396	1.388	1.445

4DLtd02	# cells	# obs	cell-wts	obs-wts
0	0	0	NA	NA
1.5	9	537	1.098	1.152
3	12	702	1.109	1.134
5	12	702	1.125	1.153
8.5	12	702	1.245	1.275

Notes on Table 1. Each of the 23 panels reports cross-entropy results for a Ford Windstar model-year ('95-'98) or variety within a model-year ('99-'02). For model-years '99-'02, varieties are sorted, left to right, in order of increasing relative as-if-new price (so the nicer varieties are on the right). Test values for a (i.e., 0, 1.5, 3, 5, 8.5) are given in each panel's leftmost column. The numbers of cells for which estimation converged, for the test value for a, are shown in the first boxed column of each panel, and the numbers of transactions in those cells are shown in the second boxed column. Cell-count weighted averages of converged cells' cross-entropies vis-à-vis the priors constitute the third boxed column, and transaction-count weighted averages make up the fourth. The yellow shading highlights the values of a with the smallest cross-entropy by the two weighting schemes.

Table 2: Preferred *r*-Specifications for Each Calendar-Quarter

1997:1	# cells	# obs	cell-wts	obs-wts
0.12	4	124	1.375	1.375
0.15	4	124	1.126	1.126
0.18	3	93	1.098	1.098

1997:2	# cells	# obs	cell-wts	obs-wts
0.12	0	0	NA	NA
0.15	1	415	0.764	0.764
0.18	0	0	NA	NA

1997:3	# cells	# obs	cell-wts	obs-wts
0.12	7	3168	1.137	1.108
0.15	8	3732	1.156	1.169
0.18	6	2799	1.128	1.141

1997:4	# cells	# obs	cell-wts	obs-wts
0.12	7	1759	1.122	1.121
0.15	8	2120	1.199	1.269
0.18	6	1590	1.176	1.245

1998:1	# cells	# obs	cell-wts	obs-wts
0.12	9	2532	0.857	0.855
0.15	13	4151	1.077	1.099
0.18	10	3182	1.042	1.061

1998:2	# cells	# obs	cell-wts	obs-wts
0.12	20	4977	1.191	1.109
0.15	22	5468	1.272	1.276
0.18	18	4578	1.232	1.221

1998:3	# cells	# obs	cell-wts	obs-wts
0.12	30	8470	1.319	1.302
0.15	30	8916	1.196	1.176
0.18	22	6082	1.179	1.173

1998:4	# cells	# obs	cell-wts	obs-wts
0.12	32	8400	1.153	1.233
0.15	37	9216	1.123	1.131
0.18	30	7656	1.081	1.075

1999:1	# cells	# obs	cell-wts	obs-wts
0.12	40	12168	1.080	1.074
0.15	39	11520	1.093	1.101
0.18	34	10559	1.376	1.792

1999:2	# cells	# obs	cell-wts	obs-wts
0.12	44	15134	1.122	1.143
0.15	45	15976	1.089	1.099
0.18	36	12374	1.067	1.093

1999:3	# cells	# obs	cell-wts	obs-wts
0.12	50	17290	1.173	1.163
0.15	47	16009	1.135	1.119
0.18	41	13904	1.105	1.090

1999:4	# cells	# obs	cell-wts	obs-wts
0.12	60	15364	1.109	1.064
0.15	59	14755	1.090	1.036
0.18	52	13082	1.104	1.066

2000:1	# cells	# obs	cell-wts	obs-wts
0.12	70	24196	1.088	1.063
0.15	66	22434	1.066	1.037
0.18	61	20152	1.105	1.074

2000:2	# cells	# obs	cell-wts	obs-wts
0.12	69	24186	1.045	1.014
0.15	72	24952	1.066	1.043
0.18	66	22393	1.093	1.063

2000:3	# cells	# obs	cell-wts	obs-wts
0.12	74	23921	0.992	1.025
0.15	78	27139	1.022	0.982
0.18	72	24083	1.144	1.111

2000:4	# cells	# obs	cell-wts	obs-wts
0.12	80	22862	0.899	0.911
0.15	79	21737	0.944	0.924
0.18	74	21432	0.984	0.970

2001:1	# cells	# obs	cell-wts	obs-wts
0.12	84	22186	1.022	1.019
0.15	86	22991	1.087	1.078
0.18	80	20656	1.123	1.103

2001:2	# cells	# obs	cell-wts	obs-wts
0.12	88	25653	1.042	1.035
0.15	90	26979	1.106	1.105
0.18	85	25251	1.176	1.176

2001:3	# cells	# obs	cell-wts	obs-wts
0.12	105	30568	1.020	1.009
0.15	103	29602	1.078	1.065
0.18	99	28519	1.111	1.095

2001:4	# cells	# obs	cell-wts	obs-wts
0.12	110	35575	1.070	1.051
0.15	108	34408	1.139	1.118
0.18	104	33021	1.189	1.160

2002:1	# cells	# obs	cell-wts	obs-wts
0.12	139	49400	1.184	1.223
0.15	143	54152	1.238	1.261
0.18	136	50342	1.282	1.288

2002:2	# cells	# obs	cell-wts	obs-wts
0.12	144	56529	1.087	1.073
0.15	145	55615	1.157	1.158
0.18	139	53781	1.206	1.211

2002:3	# cells	# obs	cell-wts	obs-wts
0.12	168	72353	1.083	1.050
0.15	172	71877	1.148	1.121
0.18	166	71635	1.185	1.169

2002:4	# cells	# obs	cell-wts	obs-wts
0.12	187	53695	1.036	1.031
0.15	187	54669	1.105	1.096
0.18	182	50558	1.182	1.163

2003:1	# cells	# obs	cell-wts	obs-wts
0.12	185	70221	1.001	0.980
0.15	189	71647	1.068	1.029
0.18	180	67802	1.103	1.076

2003:2	# cells	# obs	cell-wts	obs-wts
0.12	210	79352	1.043	1.046
0.15	207	80371	1.086	1.095
0.18	200	73762	1.137	1.150

2003:3	# cells	# obs	cell-wts	obs-wts
0.12	238	82837	1.097	1.092
0.15	241	84338	1.169	1.164
0.18	235	82620	1.351	1.457

2003:4	# cells	# obs	cell-wts	obs-wts
0.12	269	61235	1.013	1.027
0.15	272	60939	1.067	1.080
0.18	268	59808	1.134	1.149

2004:1	# cells	# obs	cell-wts	obs-wts
0.12	281	77133	1.025	1.006
0.15	288	80942	1.083	1.067
0.18	284	77885	1.140	1.130

2004:2	# cells	# obs	cell-wts	obs-wts
0.12	303	66945	1.083	1.077
0.15	307	67923	1.142	1.135
0.18	300	66263	1.211	1.211

2004:3	# cells	# obs	cell-wts	obs-wts
0.12	322	64100	1.079	1.071
0.15	331	64886	1.147	1.140
0.18	318	62743	1.206	1.199

2004:4	# cells	# obs	cell-wts	obs-wts
0.12	323	54680	0.962	0.968
0.15	319	54161	1.015	1.029
0.18	317	53316	1.082	1.087

2005:1	# cells	# obs	cell-wts	obs-wts
0.12	335	65813	1.009	0.979
0.15	334	65380	1.056	1.028
0.18	330	65087	1.150	1.104

2005:2	# cells	# obs	cell-wts	obs-wts
0.12	357	68003	1.001	0.971
0.15	354	66929	1.042	1.018
0.18	347	65311	1.103	1.078

2005:3	# cells	# obs	cell-wts	obs-wts
0.12	373	67589	0.961	0.927
0.15	376	67955	1.017	0.982
0.18	372	66547	1.109	1.053

2005:4	# cells	# obs	cell-wts	obs-wts
0.12	370	57086	0.795	0.758
0.15	372	57164	0.833	0.799
0.18	367	56269	0.884	0.853

2006:1	# cells	# obs	cell-wts	obs-wts
0.12	378	64041	0.874	0.824
0.15	385	64747	0.888	0.842
0.18	381	63769	0.922	0.876

2006:2	# cells	# obs	cell-wts	obs-wts
0.12	352	53846	0.819	0.759
0.15	352	53890	0.831	0.781
0.18	347	53022	0.863	0.817

2006:3	# cells	# obs	cell-wts	obs-wts
0.12	359	52182	0.798	0.753
0.15	357	52079	0.830	0.792
0.18	352	51258	0.872	0.836

2006:4	# cells	# obs	cell-wts	obs-wts
0.12	358	44331	0.760	0.710
0.15	362	44511	0.803	0.751
0.18	352	43694	0.866	0.802

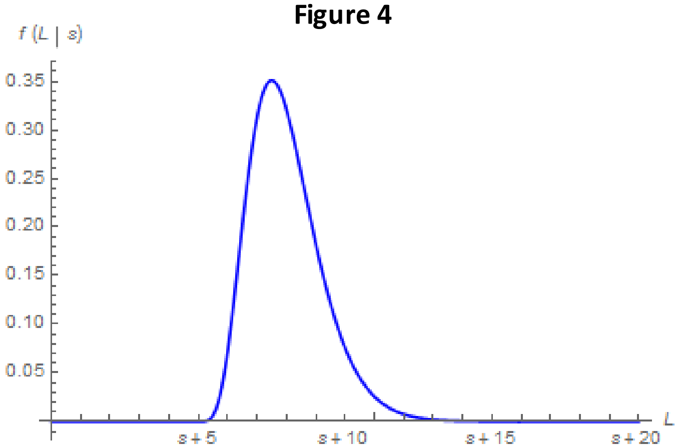
2007:1	# cells	# obs	cell-wts	obs-wts
0.12	353	48006	0.789	0.730
0.15	354	48042	0.816	0.756
0.18	350	47443	0.848	0.787

2007:2	# cells	# obs	cell-wts	obs-wts
0.12	285	29700	0.782	0.731
0.15	285	29700	0.819	0.768
0.18	278	29109	0.860	0.809

Notes on Table 2. Each of the 42 panels reports cross-entropy results for a quarter of a calendar-year in 1997-2007. Test values for r (i.e., .12, .15, .18) are given in each panel's leftmost column. The numbers of cells for which estimation converged, for the test value for r, are shown in the first boxed column of each panel, and the numbers of transactions in those cells are shown in the second boxed column. Cell-count weighted averages of converged cells' cross-entropies vis-à-vis the priors constitute the third boxed column, and transaction-count weighted averages make up the fourth. The yellow shading highlights the values of r with the smallest cross-entropy by the two weighting schemes.

To say that is a specification is "preferred" does not mean the finished probabilities resemble their priors all that well. (Specifications that aren't preferred fared even worse.) As against the increasingly truncated service-life survival-density priors plotted in Figure C.1.3 of Appendix C.1, the "typical" marginal posterior service -life density for an age-*s* cell instead resembles Figure 4, below:

Figure 4



Notes on Fig. 4. On the horizontal axis, "s" refers to most ages > 3. Windstars (and maybe other vehicles) would seem to be hauled to auction "when they have 8 years left on them." [22]

There is essentially no mass within five years of the transacted vehicle's age *irrespective of the age.* This casts doubt on the Hulten-Wykoff survivor correction to used-asset prices for motor vehicle auctions, as many survivors simply aren't present at all ages. To approximate a representative service-life density, average across the cell-specific densities, using each cell's share — i.e., $f(s,t)$ — of the cohort's transactions as weights:

$$f(L) = \sum_s \sum_t f(L|\, s,t)\; f(s,t) \tag{4.1}$$

This rolling up of the conditional densities works as long as the conditioning density, $f(s,t)$, is in some sense representative.[23] I carried out the exercise for each of the 23 varieties at its preferred a-value, using the preferred r-value of each calendar-quarter and dropping any cells that did not converge.[24] Table 3, below, gives first and second moments for each of the implied marginal service-life distributions. Plots of the implied (numerical) densities nearly all had similar shapes: a sharp rise near age 5, a peak in the range of 10 to 13 years, and then a tapering right tail. The densities resembled the Extreme Value distribution:

$$f(L) \approx Exp\left(\frac{\alpha-L}{\beta} - e^{\frac{\alpha-L}{\beta}}\right)/\beta \tag{4.2}$$

so I matched moments against the Extreme Value mean ($\alpha + \gamma\,\beta$) and variance ($\pi^2\beta^2/6$) to generate the α and β columns.[25] I also averaged across varieties' means (using transaction-count weights) and variances (by squared transaction-count weights) to generate an overall mean service-life (12.81 years) and variance (7.83 years^2). The

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[22] An examination of over 200 plots of '95 Windstar marginal posterior service-life densities *versus* their priors confirmed this shape is representative of nearly all cells age 5 and above. Cells nearer 3 years old often showed bimodal densities.

[23] Transactions of vehicles less than 3 years old, dominated by floods of manufacturers' and fleet auctions, are plainly not representative. Dropping such transactions amounts to setting $f(s{<}3,t) = 0$ and cutting off any retirements below age 3. This is severe, but not even R.L. Polk catches many automotive retirements before then.

[24] Such global a- and r- values are often different from the cell-specific preferred values. (I banked all the converged cells' specifications' resultant as-if-new and miles-and-lives densities for subsequent use.) I did not impute the service-life densities of dropped cells (say, as averages of the densities of neighboring cells), though that remains a possibility.

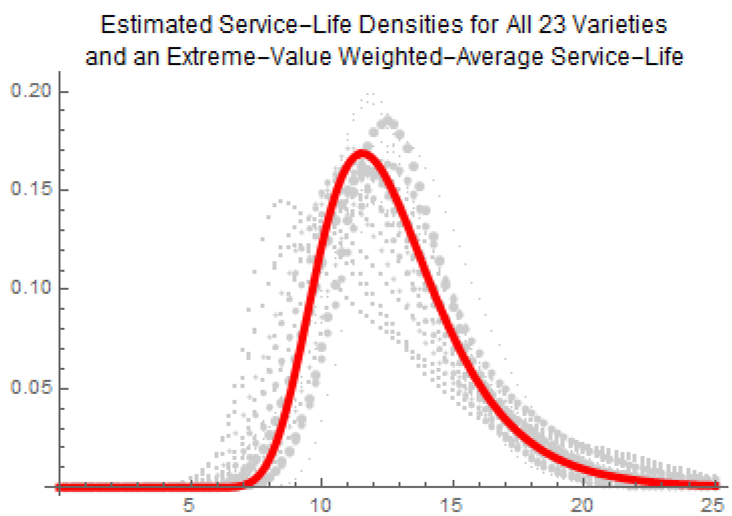
[25] Euler's Gamma constant $\gamma \approx 0.577216$.

Table 3: Preferred Service-Life Distributions

variety	# obs	mean	variance	α	β
95	62530	13.353	6.928	12.168	2.052
96	44364	13.867	9.311	12.494	2.379
97	8434	13.713	10.024	12.288	2.469
98	71105	12.758	8.247	11.465	2.239
30L99	1921	13.058	10.011	11.634	2.467
LX99	33227	12.120	6.257	10.995	1.950
SE99	10549	13.137	6.686	11.973	2.016
SEL99	2435	12.357	4.752	11.376	1.700
Base00	2678	12.750	14.010	11.065	2.918
LX00	27276	11.871	7.256	10.658	2.100
SE00	11657	12.127	6.823	10.952	2.037
SELLtd00	10784	12.916	8.196	11.627	2.232
3DLX01	3701	11.330	12.296	9.752	2.734
4DLX01	14141	11.744	9.193	10.379	2.364
SESport01	2298	12.380	7.991	11.108	2.204
SE01	5531	12.491	8.269	11.196	2.242
SEL01	2313	12.941	6.671	11.778	2.014
Ltd01	193	12.465	5.770	11.384	1.873
3DLX02	2606	12.361	17.671	10.470	3.278
4DLX02	6546	11.892	13.242	10.255	2.837
4DSE02	4316	12.043	8.604	10.723	2.287
4DSEL02	1488	12.989	6.723	11.822	2.022
4DLtd02	234	14.008	5.584	12.945	1.842

overall Extreme Value density (α = 11.548, β = 2.181) is plotted in red in Figure 5, over a background of discrete plots of the twenty-three varieties' numeric densities:

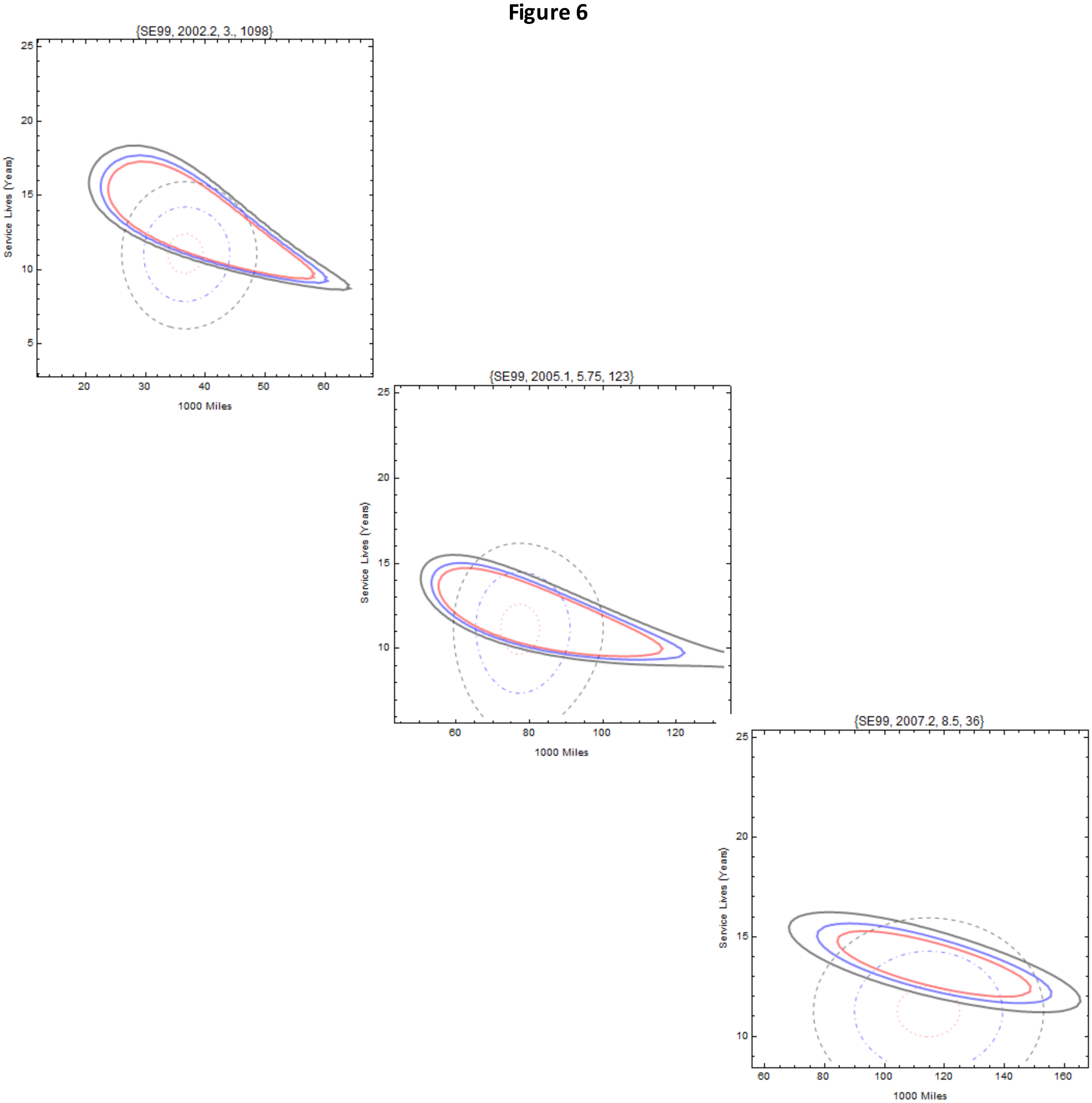
Figure 5



Notes on Fig. 5. Background dot-plots are at quarterly frequency scaled up by four. The areas of the gray dots are proportional to the number of observations of each variety.

The evidence of age-specific marginal service-life densities (instead of the same density with different degrees of left-truncation) may explain something about the behavior of the tradeoff between service-lives and miles driven. "Everyone knows" that driving more reduces the service-life (except when I selected independent priors for miles and lives), as the following three plots — for say, the youngest available (age 3), middling (5.75), and oldest available (8.5) converged cells of the SE99 variety (for a = 1.5, r = .15) — confirm:

Figure 6



Notes on Fig. 6. The red inner, blue middle, and black outer contour plots contain 50, 75, and 95 percent of the bivariate miles-and-lives probability mass. The corresponding dotted ovals show contours of the independent priors. Each plot's title-code gives the calendar-quarter, minimum quarterly age, and number of observations.

Moreover, the service-life penalty is more severe for the youngest cell (roughly 8 weeks for driving another 1000 miles), than for the middle cell (almost 3 weeks), or the oldest (2 weeks). Yet the approximate range of service-lives in the latter two cells is about the same. After nearly three more years of being driven, why aren't the vans that appeared in the middle-age auction showing up with substantially degraded service-lives in the old-age auction? The answer is: they aren't. The vehicles that were traded at the middle auction aren't worth hauling to the old auction, and the vans present at the old-age auction were still good enough in middle age for their owners to keep them.

Remember the theory of the user cost was motivated by the argued-for fluidity of used-capital resale markets (Hall and Jorgenson, 1963), and I have adhered to that theory in the development of individual- and cohort- level age-efficiency, resale-price, and rental-price profiles. Yet in this very deep and most fluid of used-capital markets, only a moveable subset of the available assets are actively traded at any given time. Are they enough to equilibrate rental prices vis-à-vis resale prices?

5. Conclusions

The trouble with good data and a careful model to assess it is that durable understandings get upset. This paper and its appendixes have provided evidence for several upsets. First, depreciation can be split into wear-and-tear, which operates on individual vans, and obsolescence, which operates across cohorts. Combined into the model-year, the rate of depreciation of Windstars as an asset-type is not even approximately constant. As the growth rate of investment in light vehicles is not constant either (Diewert and Wei, 2017), the case for vintage aggregation of motor vehicle capital is strong. Moreover, there are cross-consistent functional forms on the productive and wealth sides to make vintage aggregation of capital tractable. The shape the BLS has long used to gauge the impact of individual-level motor vehicle capital on the production side looks about right, though that agency's weights across individuals aren't correct and it neglects obsolescence.

Second, the moments of transaction-level resale-price and miles data can be put to good use estimating localized service-life densities alongside the parameters of wear-and-tear depreciation (and prior to that, of inflation and obsolescence). Plausible distributions need no longer be imported to correct for survivor-bias in age-price regressions. Moreover, Hulten-Wykoff survivorship corrections themselves are likely inadequate.

Finally, business (i.e., residual) income in the National Accounts could stand to be split into anticipated and unanticipated components, intimately connected to the own rate of return.

It remains to find sponsors to encourage this work's implementation.

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Appendix A. Age-Efficiency and Resale-Price Profiles: Consistency in Aggregation at the Cohort Level

Section 1 demonstrated the mutual consistency of the individual-level age-efficiency (1.1), resale-price (1.2), and rental-price (1.3) profiles. Then it embedded the individual-level resale-price profile within a service-life density (1.4) and contrasted the resulting aggregate to the survivors' aggregate (1.5). Finally, softening the Law of One Price to statistical independence between the distributions of as-if-new prices and service-lives, the discussion concluded with (1.6), which set the pattern for statistical estimation by moment-matching.

To use the results of estimation in national accounting and productivity analysis, which are conducted at the cohort and net-stock levels of aggregation, one must show that the cohort-level analogs of (1.1), (1.2), and (1.3) are mutually consistent also. This appendix carries out the necessary exercises. Most of the work is done without reference to movements in as-if-new prices, which are restricted to be independent of the service-life distribution(s). Average as-if-new prices are then brought back into the mix once the basic aggregates are set.

To start, note that the cohort-level rental-price profile $\mathrm{P}(s)$ uses the same service-life density, *f*(*L*), as the cohort-level resale-price profile[26]:

$$\mathrm{P}(s) = r\Theta(s) - \partial\Theta(s)/\partial s = \int_s^{\infty} f(L)\{r\,\theta(s,L) - \partial\theta(s,L)/\partial s\}\,dL \qquad \text{(A.1)}$$
$$= \int_s^{\infty} f(L)\frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}\frac{e^{as/L}-e^{a}}{1-e^{a}}dL = \int_s^{\infty} f(L)\,\rho(s,L)dL$$

Evaluating the cohort rental profile at age 0 and multiplying by P^0_{vt} would give the cohort-level user-cost if the Law of One Price for (as-if-) new assets held strictly: $P^0_{vt}\times\mathrm{P}(0)$. Otherwise use $\int_{\underline{P^o_{v,t}}}^{\overline{P^o_{v,t}}} g\left(P^o_{v,t}\right) P^o_{v,t}\, dP^o_{v,t} \ \times \mathrm{P}(0)$.

Dividing cohort-level rents at age s by rents at age 0 returns the cohort-level age-efficiency profile (before realized obsolescence):

$$\Phi(s) = \frac{\mathrm{P}(s)}{\mathrm{P}(0)} = \frac{\int_s^{\infty} f(L)\frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}\frac{e^{a\,s/L}-e^{a}}{1-e^{a}}dL}{\int_0^{\infty} f(L)\frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}dL} = \int_s^{\infty}\left\{\frac{f(L)\frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}}{\int_0^{\infty} f(L)\frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL})-rL(1-e^{-a})}dL}\right\}\frac{e^{a\,s/L}-e^{a}}{1-e^{a}}\,dL \qquad \text{(A.2)}$$

The numerator integral's domain ($L \geq s$) is narrower than that of the denominator integral (i.e., $L \geq 0$), as zero-productivity members no longer contribute to the aggregate's service-flows.[27] Rearranging slightly shows the cohort-level age-efficiency profile to be a proper weighted average of the surviving individual profiles. However, the weights are a compound of the service-life density *f*(*L*) and the individual-level user-costs. As the latter are essentially reciprocal functions of *L*, the compound weights are front-loaded relative to *f*(*L*).

[26] The second equal-sign is valid if $\partial\Theta(s)/\partial s = \int_s^{\infty} f(L)\ \partial\theta(s,L)/\partial s\, dL$, which requires $f(L)\times\theta(s, L) = 0$ when L evaluates at s. The trapped geometric individual resale-price profile does evaluate to 0 for $L \geq s$, so the requirement is satisfied so long as $\lim_{L\to s} f(L) < \infty$. The third equal-sign follows by substitution from (1.3).

[27] *Cf.* equation (1.4), above.

By contrast, the cohort aggregation scheme used by BLS, ERS, ABS, and BoK amounts to:[28]

$$\breve{\Phi}(s) = \int_s^\infty f(L)\, \frac{e^{a\, s/L} - e^{a}}{1 - e^{a}}\, dL \tag{A.3}$$

By frontloading, (A.2) is more (downwardly) convex than (A.3). Yet (A.2) is the correct schedule, in that the same steps used to derive the individual resale-price profile from the individual age-efficiency profile, can be used to derive the cohort resale-price profile (1.4) from the cohort efficiency profile:

$$\Theta(s) \overset{?}{=} \frac{\int_s^\infty e^{-r(u-s)}\, \Phi(u)\, du}{\int_0^\infty e^{-r(u-0)}\, \Phi(u)\, du} = \frac{\int_s^\infty e^{-r(u-s)} \int_u^\infty \left\{ \dfrac{f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})}}{\int_0^\infty f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})}\, dL} \right\} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, dL\, du}{\int_0^\infty e^{-r(u-0)} \int_u^\infty \left\{ \dfrac{f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})}}{\int_0^\infty f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})}\, dL} \right\} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, dL\, du} \tag{A.4}$$

The already-evaluated common integrals cancel from the large numerator and denominator:

$$= \frac{\int_s^\infty e^{-r(u-s)} \int_u^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \right\} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, dL\, du}{\int_0^\infty e^{-r(u-0)} \int_u^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \right\} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, dL\, du} \tag{A.5}$$

The orders of integration are exchangeable (just be sure $s \le L$):

$$= \frac{\int_s^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \int_s^L e^{-r(u-s)} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, du \right\} dL}{\int_0^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \int_0^L e^{-r(u-0)} \frac{e^{a\, u/L} - e^{a}}{1-e^{a}}\, du \right\} dL} \tag{A.6}$$

The u-integrals evaluate as:

$$= \frac{\int_s^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \times \frac{a\left(1-e^{r(s-L)}\right) - rL\left(1-e^{a(s/L-1)}\right)}{r(a-rL)(1-e^{-a})} \right\} dL}{\int_0^\infty \left\{ f(L) \frac{r(a-rL)(1-e^{-a})}{a(1-e^{-rL}) - rL(1-e^{-a})} \times \frac{a\left(1-e^{-rL}\right) - rL(1-e^{-a})}{r(a-rL)(1-e^{-a})} \right\} dL} \tag{A.7}$$

After cancellations, the expression reduces to:

$$\Theta(s) = \frac{\int_s^\infty f(L) \frac{a\left(1-e^{r(s-L)}\right) - rL\left(1-e^{a(s/L-1)}\right)}{a(1-e^{-rL}) - rL(1-e^{-a})}\, dL}{1} \tag{A.8}$$

This is indeed (1.4), so the cohort system is sound. By contrast, one cannot arrive at (1.4) from (A.3).

To use the cohort-level resale-price profile (1.4) or age-efficiency profile (A.2) in the perpetual inventories that comprise the real net wealth and productive stocks for an asset-type, respectively, multiply (1.4) or (A.2) by the ratio of the (average) as-if-new price for the vintage-v cohort that prevails at date t (that is, when the cohort is $s = t - v$ years old), to the (average) genuinely new price of the vintage-t cohort. Adapting the notation of (1.6), the price ratio, which tracks realized obsolescence, is:

[28] These agencies use hyperbolic not trapped-geometric individual efficiency profiles, and they top-limit their service-lives, but such differences are secondary to the basic question of proper weighting.

$$\frac{E\left(P_{v,t}^{0}\right)}{E\left(P_{t,t}^{0}\right)} = \frac{\int_{\underline{P_{v,t}^{o}}}^{\overline{P_{v,t}^{o}}} g\left(P_{v,t}^{o}\right) P_{v,t}^{o}\, dP_{v,t}^{o}}{\int_{\underline{P_{t,t}^{o}}}^{\overline{P_{t,t}^{o}}} g\left(P_{t,t}^{o}\right) P_{t,t}^{o}\, dP_{t,t}^{o}} \tag{A.9}$$

So the weight on the count of installed units from vintage *v* in the date-*t* real wealth stock is $\frac{E\left(P_{v,t}^{0}\right)}{E\left(P_{t,t}^{0}\right)}\,\Theta(s)$, and the weight in the real productive stock is $\frac{E\left(P_{v,t}^{0}\right)}{E\left(P_{t,t}^{0}\right)}\,\Phi(s)$. Both weights equal 1 for the frontier vintage (that is, for *v* = *t* and *s* = 0). Originally installed units that have been retired remain in the aggregates (as zeros), as both Θ(s) and Φ(s) account for them. Observe that while Θ(s) and Φ(s) are *time-series* measures of depreciation and deterioration, respectively (albeit expressed in terms of of age), multiplying them by the obsolescence-ratio $E\left(P_{v,t}^{0}\right)/E\left(P_{t,t}^{0}\right)$ makes them cross-sectionally safe to use in proper net-stock aggregates.

Appendix B. Data Work

Of the Raw Auction Data Set's 78,906,355 transactions, some 1,005,663 were of Ford Windstars (model-years '95-'03) and Freestars ('04-'07) and Mercury Montereys ('04-'07). A total of 2,668,487 Windstars, Freestars, and Montereys were built at Ford's Oakville, Ontario, assembly plant (and nowhere else) between January 31, 1994 and November 17, 2006 — the end of the line — so Windstar/Freestar/Monterey transactions in the dataset probably represent a solid third of the units produced, even after adjusting for possible repeat sales.[29] Earlier model years have had more time to enter the auction data, so they are better represented than later model years (i.e., model years '95-'02, the focus of this paper, accounted for 847,520 transactions).

The missing key to these data is a reliable measure of each van's age. Contrary to folk wisdom, the masked digits 12-17 of the Vehicle Identification Number do not encode build-dates. Nonetheless, digits 13-17 are required by law to be an increasing numeric sequence that corresponds to the build order (as the car-maker defines it), so sorting the data sequentially by the masked digits, making exceptions for particular sub-groupings of engine and transmission types, trim level, or foreign market (often flagged by digit 12), and then linking the sorted vehicles to an external source of cumulative builds by date, is the surest route to estimating individual births, short of VIN-specific manufacturers' records or states' initial registration dates.

At my request, NADA *did* sort the records by country of origin, assembly plant code (VIN digit 11), digits 12-17 before masking (with a flag for whether digit 12 was a letter), and auction date. I was able to confirm NADA's procedure both by a visit to the Association's Tysons, Virginia headquarters in August 2007, and by comparing the masked file to some 100,000 *unmasked* records of transactions from July 16-20, 2001, which NADA had sent me years earlier for free, before the Association became so solicitous of privacy. The sort also enabled estimates of probable repeat sales of the same vehicle (i.e., consecutive transactions having the same

[29] Source: http://infobank.wardsauto.com for build counts (password protected) and *Ward's Automotive Yearbook* (various years) for start-up and build-out dates. Wikipedia's model-year changeover dates differ from Ward's by a couple weeks.

visible portion of the VIN — among which the check-digit — nondecreasing miles, and an increasing date-of-sale). Repeats comprised perhaps one sale in eleven.

My guiding assumption was that vehicles at auction, sorted as specified and adjusted for probable repeat sales of the same vehicle, fairly represented the order and density in which they were built, at least for model-years that were present in resale markets long enough to mix well.[30] Then it was easy to match quantiles of sorted, unique vehicles at auctions to the quantiles of build-counts presented weekly or monthly by assembly plant (even by line) at Ward's http://infobank.wardsauto.com between start-up and build-out dates published in *Ward's Automotive Yearbook*. The chief complication is when vehicles with different trim-levels, engine-transmission combinations, or intended markets are built at the same plant or line but have sharply different VINs (often indicated by VIN-digit 12). Here the thousands of full VINs listed at http://www.decodethis.com enabled mapping VIN-digit 12-17 zones for various vehicle carve-outs, which I then assumed were built in parallel.[31,32] Fortunately, Ford's Oakville plant had no special VIN carve-outs, so assigning Windstars, Freestars, and Montereys build-dates accurate to the nearest few weeks was straightforward. (Different Windstar varieties were indistinguishable in the VINs of model-years '95 – '98. Model-years '99 and 2000 each had four passenger varieties that could be tracked separately, model-year '01 had six, and '02 had five.)

To prepare the dated data for use, I removed auction transactions from Canada, the Caribbean, and "Unknown" regions as a precaution against unreliable sales-price dollar-values. I pulled out any transactions that were probably the first of a pair of repeat-sales of the same vehicle if the dates of sale were within 10 weeks of each other, out of concern that the first sale had been undervalued. (This is particularly worrisome for mass sell-offs by manufacturers at the end of the model year or by fleets at the end of a leasing period, typically near age 3.) I organized what remained into quarter-year age × quarter-year date "cells" for each of the 23 identifiable varieties. The vehicles in each cell are as homogeneous as I can group them, differing substantially only in accordance with the unobserved-but-estimable distribution of their as-if-new prices and the joint distribution of their observed odometer readings and unobserved-but-estimable service-lives. Then I purged outliers, transactions too far from their relevant joint price×miles centers, by two robust techniques: removing observations that were more than 8 lengths of Rousseeuw and Croux's Q_n (1993) (instead of a standard deviation) away from the minimizer of a robust Mahalanobis distance (Gnanadesikan and J.R Kettenring, 1972) (instead of a mean or spatial median), or that surpassed 95-99 percent of the observations (depending on the number of transactions in a cell) mapped by *Mathematica*'s canned nonparametric kernel density routine. "Eyeballing" caught few more outliers. Several very young cells, which contained observations on vehicles less than half a year old, had more "outliers" (i.e., vehicles with next-to-no miles at all) than regular observations: I reserved these for direct tests of as-if-new prices. I also dropped cargo vans, and vehicles at least three years old with fewer than a thousand miles: if a van which is no longer new is not driven, something is wrong. After all

[30] I ruled out the '07 model-year out of concern that it hadn't enough time to mixed well by BEA's purchase.

[31] This is a rough approximation, probably off by a few weeks, of what was probably the actual behavior: building batches of *identical* vehicles for a week or two before turning to the next trim for its turn. The alternative, building in series — e.g., assembling all the sedans, then all the wagons, then all the coupes, over the course of a whole year — is not believable.

[32] The long, staggered-uniform VIN12-17 sequences from http://www.decodethis.com/ also permitted solving the "German tank problem" (Ruggles and Brodie, 1947) to estimate vintage/variety total production in cases where Ward's *Yearbook* neglected build-out / start-up transitions. That way I could estimate when missing model-year changeovers happened.

these procedures, I then dropped any age $\times$ date cells left with fewer than 29 observations.[33] All the various rounds of cleaning left 856,962 usable transactions, of which 724,992 reside in model years '95-'02. Finally, since the statistical model to be applied depends on auction participants being able to estimate individual service-lives with some precision, I dropped all cells less than three years old.[34] This was expensive, for it knocked out nearly all manufacturers' and many fleet auction transactions. At the end of all the cleaning, I kept the following counts of observations for each variety, organized into cells:

Table B.1

Model-Year/Variety	obs	cells	Model-Year/Variety	obs	cells
'95 (all passenger)	67,753	259	'01: 3D LX	4,235	52
'96 (all passenger)	50,735	200	'01: 4D LX	16,276	69
'97 (all passenger)	8,985	61	'01: SE Sport	2,661	34
'98 (all passenger)	79,369	189	'01: SE	5,713	50
'99: 3.0-liter	3,381	43	'01: SEL	2,942	41
'99: LX	40,444	88	'01: Ltd	193	4
'99: SE	10,807	89	'02: 3D LX	3,378	38
'99: SEL	2,767	40	'02: 4D LX	6,579	39
'00: Base	3,592	56	'02: SE	4,688	41
'00: LX	34,028	91	'02: SEL	1,944	25
'00: SE	11,804	83	'02: Ltd	285	5
'00: SELLtd	10,937	76	**Totals:**	**373,496**	**1673**

Batteries of uncentered moments (with respect to powers of prices and miles) within each cell constitute the objects to be matched by the minimum cross-entropy model in the estimation section.

[33] In retrospect, a minimum-count threshold of 50 would have been wiser, as 29 observations can barely support a mean, let alone the higher moments that the estimation section spells out. Cell sizes differed widely, from 29 up through thousands at the end of a new model-year or at the end of the standard 3-year lease term.

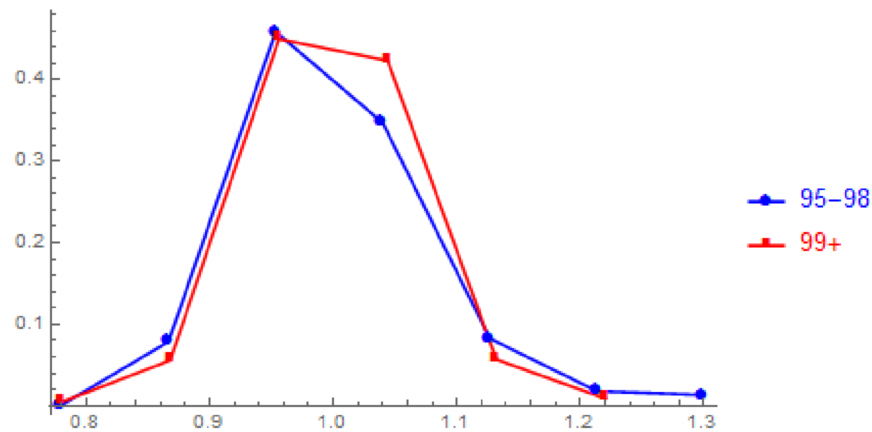
[34] A prospective buyer assessing a vehicle that isn't well individuated would fall back on the commonly-held overall density of service-lives and issue a middling bid. So would all the other bidders. The upshot is a spike of middling bids, revealing little about the service-life density that all the bidders are referencing.

Appendix C. More about Statistical Estimation

C.1 Forming Prior Distributions

To estimate prior/target densities about the as-if-new means (3.1.2), I reserved 301 prices from vans no more than a quarter-year old and driven no more than 100 miles. These are very few observations from the overall usable Windstar/Freestar sample, so I pooled, allowing individual as-if-new prices as low as 78 percent of the mean and as high as 122 percent.[35] Plots of the distributions, fit by maximum-entropy matching of the first four raw moments of prices over a coarse grid, are shown below.[36] Note these distributions are priors/targets only; subsequent cross-entropy fits may and did infer different weights on the support points. But I have imposed proportionality on the as-if-new priors across varieties and dates, so that as a variety's *mean* as-if-new price declines through time, its distribution's bounds retract proportionately, maintaining the *à priori* shape. This guarantees commonality of the own rate of return across vintages and varieties at any given date, as far as the data permit. It also shifts the burden of accounting for variations across individual prices as the cohort ages, from as-if-new prices when the cohort is young to the joint lives-and-miles distribution when the cohort is old.

Figure C.1.1



Notes on Fig. C.1.1. Prior Distributions of As-If-New Prices about Mean (Which Is Normalized to 1) Each of model-years '95-'98 uses the blue distribution, which has a somewhat longer right tail. The algebra is: $q_n \propto Exp[318.7\,n + 71.4\,n^2 - 347\,n^3 + 143\,n^4]$ *for* $n \in \{.78, .867, .953, 1.04, 1.267, 1.213, 1.3\}$*. Each variety of model-years '99-'06 uses the red distribution, which is nearly symmetric. The algebra is:* $q_n \propto Exp[-4868.4\,n + 7691.9\,n^2 - 5282.9\,n^3 + 1333\,n^4]$ *for* $n \in \{.78, .868, .956, 1.044, 1.132, 1.22\}$*. Both densities are discrete; the connecting segments are visual aids only.*

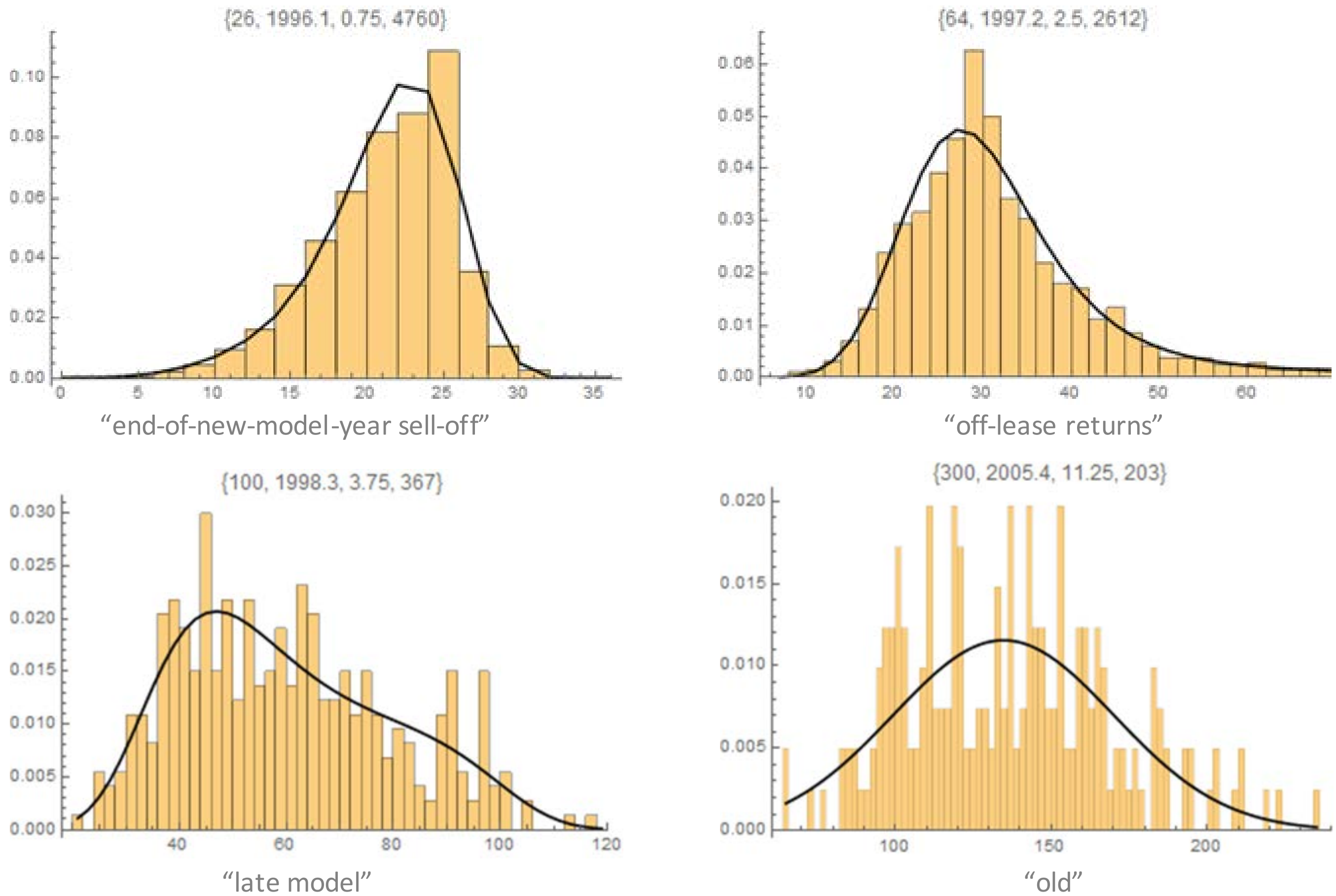
I settled for independence priors for the joint distribution of service-lives and miles. The univariate prior on observed miles is the (up-to) 4-moment maximum entropy fit (Zellner and Highfield, 1988) on a grid. For each cell, I tried a 4-moment fit first and accepted the result if the Lagrange multipliers were statistically significant; if not, I next tried a 3-moment fit (allowable within bounded domains), and then a 2-moment fit. A

[35] …for all varieties after model-year 1998, when the auction data permitted distinguishing trim levels, so symmetry is not a bad guess. But for model-year '95-'98, where the data subsumed all trims into just "the model year," I allowed individual as-if-new prices as high as 130 percent of the mean to account for a possible small share of high-end trims.

[36] Maximum entropy estimation gives the same ex post probabilities as minimum cross-entropy estimation with flat priors.

few cells offered no significant results even for 2-moment (i.e., discrete approximations to truncated Normal) fits, so I threw them out. The order (i.e., 2nd, 3rd, or 4th) of the univariate fit of miles then determined the order of the univariate specification of service-life priors (just below), as well as the order of the cross-entropy specification above at (3.2.1), in which the gray-background area represents the third-order terms and the blue-background the fourth order. Age- and date-specific miles distributions for a few iconic quarters from the career of model-year '95 (other varieties' patterns are similar) are shown in Figure C.1.2, below:

Figure C.1.2

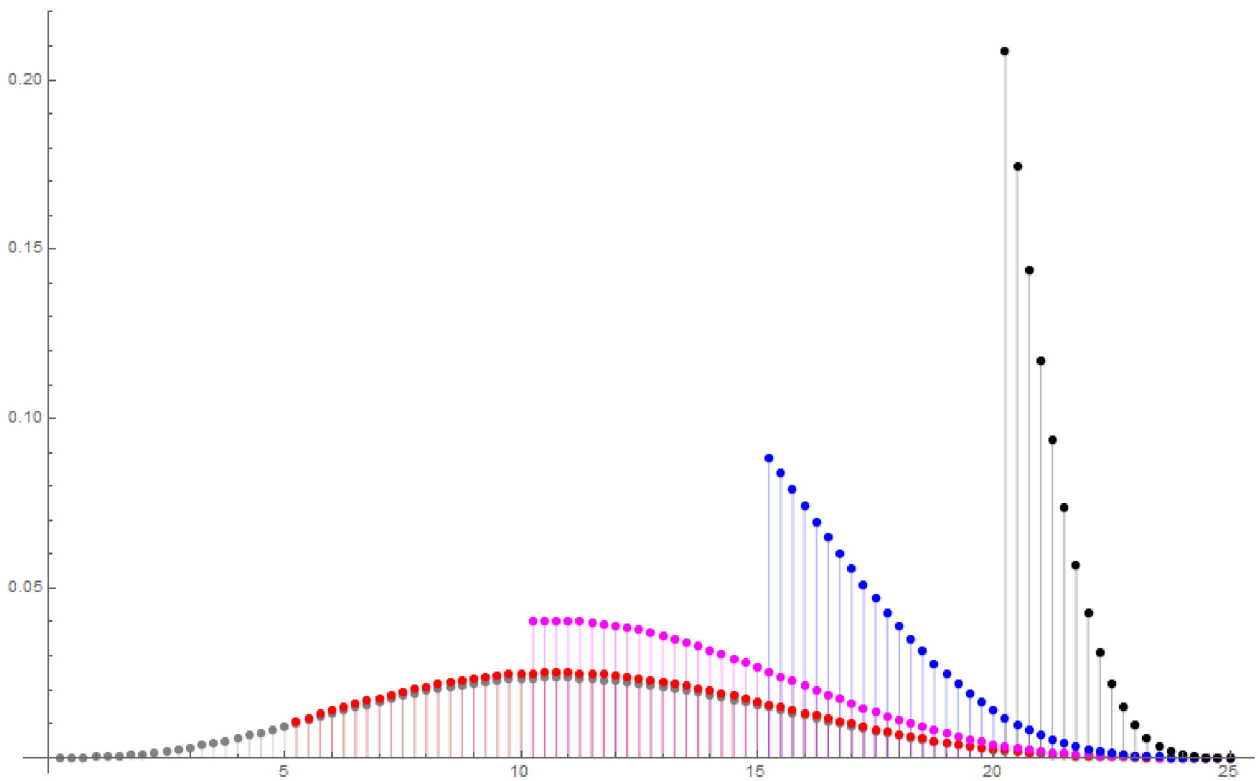


Notes on Fig. C.1.2. Histograms of Miles Driven Overlain by Maximum Entropy Fitted Densities. The top two plots show the most populated quarters from two early auction regimes: Ford dumping new vehicles near age 1 at the end of the model-year, and fleets dumping lease returns around age 3. The fitted second density doesn't do justice to the spike at 30,000 miles. The second two densities, with more typical quarterly sales volumes of 200-400 units, show dealer-dominated auction sales regimes —here near ages 4 and 11, respectively.[37] Each plot's "job code" gives its cell-number, calendar-year quarter, lower boundary of its quarterly cell-age, and number of transactions.

[37] Very young vehicles are not likely to convey much information about their individual service-lives, as buyers would spread their bets and *all* bid the (same) price of a "typical" vehicle, which biases the implied service-life density toward a spike at the value of the service-life that is consistent with the typical vehicle's price, so the 4,760 observations in the age range [.75, 1) of model-year '95, for instance, are wasted.

The univariate prior on unobserved service-lives is the (up-to) 4-moment maximum-entropy distribution that comes closest to the discrete quarter-year approximation of a progressively left-truncated Beta(4,5) density, extended to a top life of 25 years. I chose the Beta(4,5) subjectively: its untruncated mean is 11.1 years, which can't be far wrong, and its right tail runs a bit farther than its left, though actual retirements are probably more concentrated. Plots of discrete Beta densities are shown just below in Figure C.1.3 for various age-truncations.

Figure C.1.3



Notes on Fig. C.1.3. Prior Marginal Distributions of Service-Lives, Conditional on Survival to Age s. The untruncated gray curve represents my watered-down gut feeling, a compromise between a concentrated density that may better match the retirements of a narrow make of vehicles and a flat prior that amounts to the target density for maximum likelihood estimation.

C.2 Optimizing Systems That Have Independent-but-Entangled Probabilities

Here I detail the solution concept's first-order estimating equations, as well as a scheme for "pocketing" iteration-by-iteration changes in the equations' Lagrange-multiplier parameters (until further changes are minimal and the parametric expressions on one side of the equations match their empirical counterparts on the other). It turns out that the pocketing procedure is needed to circumvent what is essentially an infinite-regress problem, and so makes nearly-sure convergence feasible.

For fixed sets of prior/target probabilities $\{q_{l,m}\}$ and $\{q_n\}$, which are stored as a matrix and a vector of numbers, respectively, the (unnormalized) discrete joint probability of service-life ℓ and odometer-reading *m* is:

(C.2.1)

$$p_{l,m} \propto q_{l,m}\, Exp[\, \lambda_{P1}\overline{N1}\,\hat{P}^{o}_{v,t}\theta_l + \boldsymbol{\lambda_{M1}} m + \lambda_{P2}\overline{N2}\left(\hat{P}^{o}_{v,t}\theta_l\right)^2 + \lambda_{P1M1}\overline{N1}\,\hat{P}^{o}_{v,t}\theta_l m + \boldsymbol{\lambda_{M2}} m^2 +$$
$$\lambda_{P3}\overline{N3}\left(\hat{P}^{o}_{v,t}\theta_l\right)^3 + \lambda_{P2M1}\overline{N2}\left(\hat{P}^{o}_{v,t}\theta_l\right)^2 m + \lambda_{P1M2}\overline{N1}\,\hat{P}^{o}_{v,t}\theta_l m^2 + \boldsymbol{\lambda_{M3}} m^3 +$$
$$\lambda_{P4}\overline{N4}\left(\hat{P}^{o}_{v,t}\theta_l\right)^4 + \lambda_{P3M1}\overline{N3}\left(\hat{P}^{o}_{v,t}\theta_l\right)^3 m + \lambda_{P2M2}\overline{N2}\left(\hat{P}^{o}_{v,t}\theta_l\right)^2 m^2 + \lambda_{P1M3}\overline{N1}\,\hat{P}^{o}_{v,t}\theta_l m^3 + \boldsymbol{\lambda_{M4}} m^4]$$

and the unnormalized discrete probability of as-if-new price value *n* is:

(C.2.2)

$$p_n \propto q_n\, Exp[\boldsymbol{\lambda_{N}} n + \lambda_{P1}\overline{\theta 1}\,\hat{P}^{o}_{v,t}n + \lambda_{P2}\overline{\theta 2}\left(\hat{P}^{o}_{v,t}n\right)^2 + \lambda_{P1M1}\overline{\theta 1M1}\,\hat{P}^{o}_{v,t}n + \lambda_{P3}\overline{\theta 3}\left(\hat{P}^{o}_{v,t}n\right)^3 + \lambda_{P2M1}\overline{\theta 2M1}\left(\hat{P}^{o}_{v,t}n\right)^2$$
$$+ \lambda_{P1M2}\overline{\theta 1M2}\,\hat{P}^{o}_{v,t}n + \lambda_{P4}\overline{\theta 4}\left(\hat{P}^{o}_{v,t}n\right)^4 + \lambda_{P3M1}\overline{\theta 3M1}\left(\hat{P}^{o}_{v,t}n\right)^3 + \lambda_{P2m2}\overline{\theta 2M2}\left(\hat{P}^{o}_{v,t}n\right)^2 + \lambda_{P1M3}\overline{\theta 1M3}\,\hat{P}^{o}_{v,t}n$$

(The normalizing denominators sum over all the unnormalized $p_{l,m}$ or p_n, respectively.) The Lagrange multipliers are as in (3.2.1), but only the five that are shaded yellow here are unique to their "own" probabilities. The rest are shared, so the numerical "side moments":

(C.2.3)

$$\overline{N1} = \textstyle\sum_n p_n n \qquad \overline{N2} = \sum_n p_n n^2 \qquad \overline{N3} = \sum_n p_n n^3 \qquad \overline{N4} = \sum_n p_n n^4$$
$$\overline{\theta 1} = \textstyle\sum_l \sum_m p_{l,m}\theta_l \qquad \overline{\theta 2} = \sum_l \sum_m p_{l,m}\theta_l^2 \qquad \overline{\theta 3} = \sum_l \sum_m p_{l,m}\theta_l^3 \qquad \overline{\theta 4} = \sum_l \sum_m p_{l,m}\theta_l^4$$
$$\overline{\theta 1M1} = \textstyle\sum_l \sum_m p_{l,m}\theta_l m \qquad \overline{\theta 2M1} = \sum_l \sum_m p_{l,m}\theta_l^2 m \qquad \overline{\theta 1M2} = \sum_l \sum_m p_{l,m}\theta_l m^2$$
$$\overline{\theta 3M1} = \textstyle\sum_l \sum_m p_{l,m}\theta_l^3 m \qquad \overline{\theta 2M2} = \sum_l \sum_m p_{l,m}\theta_l^2 m^2 \qquad \overline{\theta 1M3} = \sum_l \sum_m p_{l,m}\theta_l m^3$$

reside in one set of probabilities but depend on the other.

There are as many Lagrange parameters to adjust as there are moment equalities to be satisfied. The moment equalities are[38]:

(C.2.4)

$$1 = \textstyle\sum_n p_n n \qquad 2\overline{P1} = \hat{P}^{o}_{v,t}\left(\overline{\theta 1}\sum_n p_n n + \overline{N1}\sum_l \sum_m p_{l,m}\theta_l\right) \qquad \overline{M1} = \sum_l \sum_m p_{l,m} m$$

$$2\overline{P2} = {\hat{P}^{o}_{v,t}}^{2}\left(\overline{\theta 2}\textstyle\sum_n p_n n^2 + \overline{N2}\sum_l \sum_m p_{l,m}{\theta_l}^2\right) \qquad 2\overline{P1M1} = \hat{P}^{o}_{v,t}\left(\overline{\theta 1M1}\sum_n p_n n + \overline{N1}\sum_l \sum_m p_{l,m}\theta_l m\right)$$

$$\overline{M2} = \textstyle\sum_l \sum_m p_{l,m} m^2$$

$$2\overline{P3} = {\hat{P}^{o}_{v,t}}^{3}\left(\overline{\theta 3}\textstyle\sum_n p_n n^3 + \overline{N3}\sum_l \sum_m p_{l,m}{\theta_l}^3\right) \qquad 2\overline{P2M1} = {\hat{P}^{o}_{v,t}}^{2}\left(\overline{\theta 2M1}\sum_n p_n n^2 + \overline{N2}\sum_l \sum_m p_{l,m}{\theta_l}^2 m\right)$$
$$2\overline{P1M2} = \hat{P}^{o}_{v,t}\left(\overline{\theta 1M2}\textstyle\sum_n p_n n + \overline{N1}\sum_l \sum_m p_{l,m}\theta_l m^2\right) \qquad \overline{M3} = \sum_l \sum_m p_{l,m} m^3$$

$$2\overline{P4} = {\hat{P}^{o}_{v,t}}^{4}\left(\overline{\theta 4}\textstyle\sum_n p_n n^4 + \overline{N4}\sum_l \sum_m p_{l,m}{\theta_l}^4\right) \qquad 2\overline{P3M1} = {\hat{P}^{o}_{v,t}}^{3}\left(\overline{\theta 3M1}\sum_n p_n n^3 + \overline{N3}\sum_l \sum_m p_{l,m}{\theta_l}^3 m\right)$$
$$2\overline{P2M2} = {\hat{P}^{o}_{v,t}}^{2}\left(\overline{\theta 2M2}\textstyle\sum_n p_n n^2 + \overline{N2}\sum_l \sum_m p_{l,m}{\theta_l}^2 m^2\right) \qquad 2\overline{P1M3} = \hat{P}^{o}_{v,t}\left(\overline{\theta 1M3}\sum_n p_n n + \overline{N1}\sum_l \sum_m p_{l,m}\theta_l m^3\right)$$
$$\overline{M4} = \textstyle\sum_l \sum_m p_{l,m} m^4$$

where the left-hand side empirical moments are: (C.2.5)

$$\overline{P1} = \left(\textstyle\sum_{i=1}^{nobs} price_i\right)/nobs, \qquad \overline{M1} = \left(\sum_{i=1}^{nobs} miles_i\right)/nobs,$$

[38] Not every cell has enough information to fit all 15 moments usefully. Some cells only qualify for the second-order treatment (i.e., the estimating equations and moments with only white backgrounds), others also for the third order (i.e., add the gray-background equations and moments), still others also for fourth order (i.e., add the cyan-background pieces).

$\overline{\text{P2}} = \left(\sum_{i=1}^{nobs} price_i{}^2\right)/nobs, \qquad \overline{\text{P1M1}} = \left(\sum_{i=1}^{nobs} price_i \times miles_i\right)/nobs, \qquad \overline{\text{M2}} = \left(\sum_{i=1}^{nobs} miles_i{}^2\right)/nobs,$

$\overline{\text{P3}} = \left(\sum_{i=1}^{nobs} price_i{}^3\right)/nobs, \qquad \overline{\text{P2M1}} = \left(\sum_{i=1}^{nobs} price_i{}^2 \times miles_i\right)/nobs,$
$\overline{\text{P1M2}} = \left(\sum_{i=1}^{nobs} price_i \times miles_i{}^2\right)/nobs, \qquad \overline{\text{M3}} = \left(\sum_{i=1}^{nobs} miles_i{}^3\right)/nobs,$

$\overline{\text{P4}} = \left(\sum_{i=1}^{nobs} price_i{}^4\right)/nobs, \qquad \overline{\text{P3M1}} = \left(\sum_{i=1}^{nobs} price_i{}^3 \times miles_i\right)/nobs,$
$\overline{\text{P2M2}} = \left(\sum_{i=1}^{nobs} price_i{}^2 \times miles_i{}^3\right)/nobs, \qquad \overline{\text{P1M3}} = \left(\sum_{i=1}^{nobs} price_i \times miles_i{}^3\right)/nobs,$
$\overline{\text{M4}} = \left(\sum_{i=1}^{nobs} miles_i{}^4\right)/nobs.$

Solving for the Lagrange multipliers that enact the estimating equations (C.2.4) is the last step in minimizing the constrained cross-entropy (3.2.1). I have written the estimating equations in "shorthand": $p_{l,m}$ and p_n are parameterized probabilities, with numerators (C.2.1) and (C.2.2), while $\overline{\text{N1}}$,…,$\overline{\theta\text{1M3}}$ are *numerical* side moments evaluated at the latest cross-consistent values of their own probabilities.

Estimation by some variant of the Newton-Raphson method makes sense in this medium-sized nonlinear problem. I used Dennis and Schnabel's (1996) careful line-search method, but with three tweaks.[39] First, since every proposed change to the Lagrange parameters knocked the *parametric* probabilities p_n and $p_{l,m}$ away from their thus-far matching *values* in the side moments, I developed a fixed-point procedure to restore the numerical probabilities to equality with their parametric counterparts.[40] The procedure was called every time a change in parameters was proposed, including every cut to the line-search step size. Second, the estimating equations and the Hessian matrix become much more difficult when the Lagrange parameters are different from zero — i.e., essentially, any time after the initial iteration — for then the round-and-round knock-on effects of changes in $\{\lambda\}$ must be accounted for. So I made *each* iteration the "initial" one by resetting the numerical priors $q_{l,m}$ and q_n to the resultant values of $p_{l,m}$ and p_n from the preceding iteration, while resetting all λ back to zero. This repeated "pocketing" of results-thus-far into the priors implies the ultimate parameter values are reconstructed as the sum of all the parametric revisions.[41] Third, I added a *safeguard-ridge* multiple of an identity matrix to the Hessian, to guarantee that the ratio of the largest to smallest eigenvalues of the thus-supplemented Hessian would stay within acceptable bounds: forcing the bounds too narrow (equivalently, making the ridge parameter too large) made for excruciatingly slow convergence; permitting them too wide (so the ridge too small) allowed wild parameter updates leading to irrecoverable probability spikes. As a practical matter, I set the initial eigenratio bounds to 10,000, narrowed them by a factor of 10 whenever a parameter

[39] *Cf.* Dennis and Schnabel (1996), section 6.3 especially.

[40] In words: *given* already "cross-consistent values" of the Lagrange parameters, parametric and numerical probabilities, and side moments, a change in the Lagrange values induces a change in the parametric probabilities, which in turn alter the side moments, which feed back into the *other* parametric probabilities, which in turn alter the side moments, etc. Cross-consistent values always obtained for starting values: $p_{l,m}=q_{l,m}$, $p_n=q_n$ (so the numerical probabilities within the side-moments are merely the priors), and λ=0. Although I did not formally analyze the fixed-point procedure's stability conditions, it never exploded, and its many iterations were rapid.

[41] Reconstruction via summation was adequate only. Some of the revisions happened to be subtractions, which allowed for the accretion of tiny numerical errors into small-but-annoying final ones. However, once the converged probabilities were obtained, and thus available in log form as regression dependent variables, a linear regression of $ln\,p_{l,m}$ against the de-exponentiated right-hand side of the parametric probability (with an extra parameter for the log of the normalizing denominator) became feasible. The regression polished the final Lagrange parameters of their accumulated errors well. I did not pursue the notion of deriving $ln\,p_{l,m}$ *from the start* by successive rounds of regression and renormalization.

revision reduced the cross entropy by more than 1,[42,43] and widened them by a factor of 10 whenever a full-step parameter revision reduced the cross entropy by 1/500th or less. The implied ridge parameter was interpolated along a schedule of about 200 exact eigenratio-to-ridge solutions calculated by *Mathematica* for each new computation of the Hessian.[44] Nearly all the cells that survived the data cleaning converged within 300 iterations. This paper's results are based on those that converged.

Appendix D. Applications in the National Accounts

To apply the model developed in this paper, analysts do not have to conduct high-powered econometrics, merely put together properly the results already found. BEA does not care about individual vans, but only about the net stock of motor vehicles (as well as net stocks of other assets), the net stock's depreciation, and its service-flows. So far, this paper has not dealt with the net stock as a unitary object. That is usually the province of geometric depreciation, which is BEA's default treatment for most assets, owing in part to lack of detailed data that would permit cohort-level treatments. Motor vehicles are the leading exception. We will deal with cohorts; the net stock sums over them. The example laid out in this section will be the net wealth stock of '95-'03 Ford Windstars and '04-'06 Ford Freestars and Mercury Montereys, taken as an aggregate, as well as the net *productive* stock, which is not ordinarily BEA's business. Limitations of my approach will be noted as appropriate.

Shoehorning research results into an accounting framework always provokes arguable implementation choices. The first decision concerns the treatment of (individual-into-) cohort-level resale-, rental-, and efficiency profiles. I chose to use the a-parameters separately preferred by each model-year and available variety (observation-, not cell-, weighted), rather than an overall average a-value (whether 1.5 as above, or 1.145 as an observation-weighted average), to demonstrate the flexibility of vintage accounting. Nonetheless, for varieties from model-years '03–'06, where I had already computed mean as-if-new prices but not age-price profiles embedded in service-life densities, I used a=1.145.[45] I also settled for a constant own rate of return, r = .12, across the board.[46]

The second decision was to allow each vintage and variety its own, unvarying, numeric marginal service-life density. This followed directly from allowing separate a-values. Numeric densities are easily applied as direct weights on the individual-level profiles. Moreover, the cross-entropy method's density outputs are

[42] Although "1" seems like a magic number, it did work out as the rough threshold beyond which the optimization program broke. Note that the pocketing procedure reset the cross entropy to 0 each iteration.

[43] When the eigen-ratio is reduced all the way to 1 (which never occurred), the supplemented Hessian is *only* an identity matrix, so the Newton-Raphson method reduces to gradient descent.

[44] Recalibrating the ridge factor *instead of* backtracking the line-search characterizes the "trust region" version of the Newton-Raphson method. I dabbled with trust regions without understanding them well enough to apply them to a problem that needed to pocket this round's results into next round's priors, so they too often stalled at gradient descent.

[45] An alternative might have extended the apparently increasing trend-value of a = {0, 1.516, 2.066, 2.227, 2.665} for model-years '98–'02 (observation-weighted averages of varieties) to subsequent model years.

[46] I will be happy to allow r to vary across dates when it is a credible interior solution. (However, everything in this paper has r constant across horizons flowing from any date, at whatever value happens to prevail at that date.)

already smooth, so the step of parameterizing (as, for example, to extreme-value densities) is not needed to counter noise.[47] Still, using well-known parametric densities remains an option.

A third decision was to give the distribution of miles driven — which was joint with the distribution of service-lives, in the research effort — no play in the accounts. Miles were a useful conditioning variable, but it is hard to see how to make them informative once aggregated across cells, varieties, and cohorts. Nonetheless, marginal miles driven are the closest approximation we'll ever get to a vehicle's intensive-margin service-flows, so some analytically justifiable means ought to be found to use them. That is a task for further research.[48]

The fourth set of decisions concerned the regulation of vintages. Windstars did not enjoy regular model-years. The opening '95's were built and sold for 18 months, while the '97's were relabeled '96's, sold for barely half a year without even updating the '96 VIN sequences, then dropped in favor of a year-and-a-half run of '98s. I opted to declare a model-year begun (and the until-then latest model year concluded) when a Windstar, Freestar, or Mercury Monterey with that nomenclature first appeared in my as-if-new price regression, whether or not all the varieties from that model-year had appeared yet. (See Table D.1, below.) The current model-year is important in the accounting, for the user-cost always refers to the frontier vintage, and realized obsolescence is booked relative to the frontier.[49] Nonetheless, to estimate future depreciation (i.e., after the close of the '06 model-year), I settled on a standard model-year of *cy*–1 :IV - to - *cy* :III, where "*cy*" is the calendar-year, starting at 2006:IV. Having set the frontier vintage in terms of the entire model-year rather than its most advanced (if little-bought) variety, I then scaled variety-specific resale-price and age-efficiency profiles such that their "installation-weighted average" profiles worked out to 1 at the model-year's earliest observed quarter. (This means the cheapest variety of the model-year started out below 1 while the best trim began above.) The upshot is that purchase- and rental- price indexes that aggregated across varieties within a model-year are merely unit-value indexes, where the unit is 1 van.[50] My proxy for installations — WardsAuto's unit build-counts, spread equally over the model-year's quarters — is not the last word but was easy to implement.

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[47] For model-years '95–'98, *different* service-life densities might have prevailed at different dates, according to which values of r were preferred at those dates. (Instead, I averaged across the implied preferred service-life densities from different (s,t)-cells. This is somewhat at odds with deciding r=.12 must prevail everywhere.) Densities for subsequent model-years all arose in the regime where r=.12 was indeed preferred.

[48] For instance, replace the standard production-function optimization set-up — i.e., $P_y\ f(K,\ldots) - U{\times}K - \ldots$, where P_y is the competitive price of a vehicle-user's output, f is the production function, K the productive stock of vehicles, and U their user-cost, all during period t — by one that depends explicitly on the total miles driven during period t, something such as: $P_y\ f(m{\times}K,\ldots) - (U/m)(m{\times}K) - \ldots$, where m is miles per vehicle during period t, so that $m{\times}K$ = total miles during t and U/m is the user-cost per mile. Cancellations make $(U/m)(m{\times}K)$ equivalent to $U{\times}K$, so the real change has been the replacement of K in the production function by $m{\times}K$. Now, $m{\times}K$ may fluctuate considerably over the business cycle, even if K does not. If $(\partial U/U)/(\partial m/m) \approx 1$, then a Solow productivity decomposition would see intensive-margin capital flex greatly, even as capital's share stays relatively fixed.

[49] By this rather narrow definition, the current model-year cannot be obsolete. This treats Windstar/Freestar/Montereys as an asset category unto itself, unrelated to other motor vehicles. This is not realistic, but it is as far as I can take the data. Ascertaining Windstars' position within the broader web of light vehicles would be the next step.

[50] Applying hedonic quality adjustment to the current model-year seems simple enough. But applying the new-van quality adjustment to *older* model-years, now that their as-if-new price estimates are in place, would take some rethinking.

Table D.1: First Appearance of Each Variety in My Auction Data

'95 **W**indstars 1994:3		'96W 1996:1		'97W 1997:2		'98W 1997:4
'99W:	3.0L 2000.2	LX **1999.2**		SE 1999.3	SEL 2000.2	
'00W:	Base 2001.4	LX **2000.1**		SE 2000.3	SELLtd 2000.3	
'01W:	3DLX 2001.4	4DLX **2001.2**	SESport 2001.3	SE 2002.1	SEL 2002.1	Ltd 2003.1
'02W:	3DLX 2003.3	4DLX 2002.2		4DSE **2002.1**	4DSEL 2002.4	4DLtd2002.4
'03W:	3D4D 2004.1	4DLX **2003.1**		SE 2003.2	SEL 2003.4	Ltd 2003.4
'04**F**reestars:	S 2005.1	SE 2004.3	SES **2004.2**	SEL **2004.2**		Ltd 2004.4
			'04**M**ontereys **2004.2**			
'05F:	S 2005.3	SE **2005.1**	SES 2005.2	SEL05 2005.2		Ltd 2005.2
'05M:	Convenience 2005.3		Luxury 2006.1		Premier 2006.1	
'06F:		SE **2006.1**		SEL **2006.1**		
'06M:			Luxury 2007.1			

Notes on Table D.1. Each vintage or variety first appeared in the auction data at the calendar-quarters shown. Varieties from the same vintage did not all appear together, so the earliest varietal arrival of a vintage is in bold-face. "W"=Windstar, "F"=Freestar, "M"=Monterey.

The fifth group of decisions had to do with prices. The auction data were strictly wholesale, and I have attempted no wedges up to retail value in the research or the accounting. This departs from longstanding national accounts practice.[51] Further, I have used not the auction prices themselves but my unbiased model of their mean as-if-new values (3.1.2). This fixes price-ratios across varieties and keeps rates of inflation the same among them, satisfying Hicksian aggregation. Point-values for (3.1.2), which are useful for estimating what a variety's new price would have been in calendar-periods when the variety was not actually observed, are:

Table D.2: Coefficients of PPMLE As-If-New Pricing Regression (3.1.2)

	c	2.964751377120970	β_t	94:3	1.096708395806910
				94:4	1.076171117624040
β_v	95	–1.159439220566140		95:1	1.081373113379950
	96	–1.088210621834640		95:2	1.122725937390110
	97	–1.025697901369100		95:3	1.111487700474110
	98	–0.943181933845815		95:4	1.013688134533840
	30L99	–0.974782005337930		96:1	1.000375068164110
	LX99	–0.829432750632410		96:2	1.017128055812830
	SE99	–0.712008628791034		96:3	0.973409618846769
	SEL99	–0.593354220686359		96:4	0.888970874012359
	Base00	–0.992000166265836		97:1	0.917914519998440
	LX00	–0.773727727535772		97:2	0.930125119388386
	SE00	–0.656467398496016		97:3	0.939865824513309
	SELLtd00	–0.549259015143947		97:4	0.806329244153722

[51] But maybe the national accounts are catching up. Recently, brokerage fees for residential structures have been capitalized themselves rather than incorporated into the structure values. This leaves the structures closer to wholesale.

Table D.2: Coefficients of PPMLE As-If-New Pricing Regression (3.1.2) (continued)

	3DLX01	−0.944744306117723		98:1	0.822619389709055
	4DLX01	−0.723952069408558		98:2	0.825961090345819
	SESport01	−0.635429732473724		98:3	0.861099695418847
	SE01	−0.570695102565578		98:4	0.831638714853186
	SEL01	−0.405434967733631		99:1	0.822393697773766
	Ltd01	−0.375142320497973		99:2	0.809688476298551
	3DLX02	−0.810092401767964		99:3	0.823320204454980
	4DLX02	−0.642827633065381		99:4	0.754491246562543
	4DSE02	−0.530053532368935		00:1	0.719273281871659
	4DSEL02	−0.334299513111451		00:2	0.705963352162072
	4DLtd02	−0.300426648795869		00:3	0.722653457521937
	3D4D03	−0.731842607055780		00:4	0.641399698602244
	4DLX03	−0.583090485340029		01:1	0.635380513466613
	SE03	−0.476262953854484		01:2	0.621354081443877
	SEL03	−0.272185810096986		01:3	0.623846723141716
	Ltd03	−0.228226094005815		01:4	0.558659477070338
β_v	S04	−0.621148088315499	β_t	02:1	0.595169741840899
	SE04	−0.520051992747847		02:2	0.555418147537687
	SES04	−0.465708646806284		02:3	0.520862254159833
	SEL04	−0.387658212311193		02:4	0.398250377254785
	Ltd04	−0.226603933619934		03:1	0.343859846518590
	M04	−0.398205015378774		03:2	0.364102262530881
	S05	−0.492821816842997		03:3	0.407064387399965
	SE05	−0.429183037140144		03:4	0.302154800240477
	SES05	−0.385610468464100		04:1	0.282385942163401
	SEL05	−0.294531345783530		04:2	0.318099508759867
	Ltd05	−0.096578022794287		04:3	0.317153913309634
	M05	−0.280816008366816		04:4	0.173315299401982
	MLuxury05	−0.082660196167296		05:1	0.208210623776514
	MPremier05	0.036922717014200		05:2	0.223176922258330
	SE06	−0.351346602765633		05:3	0.186940565061641
	SEL06	−0.229559639702223		05:4	0.045494133890147
	MLuxury06	"0"		06:1	0.007911871767545
				06:2	−0.001504634937432
				06:3	−0.004084377948703
				06:4	−0.096635232076967
				07:1	−0.098891780673675
				07:2	"0"

Moreover, for purposes of out-of-sample forecasts, I fit trend regressions of the point-values for time:

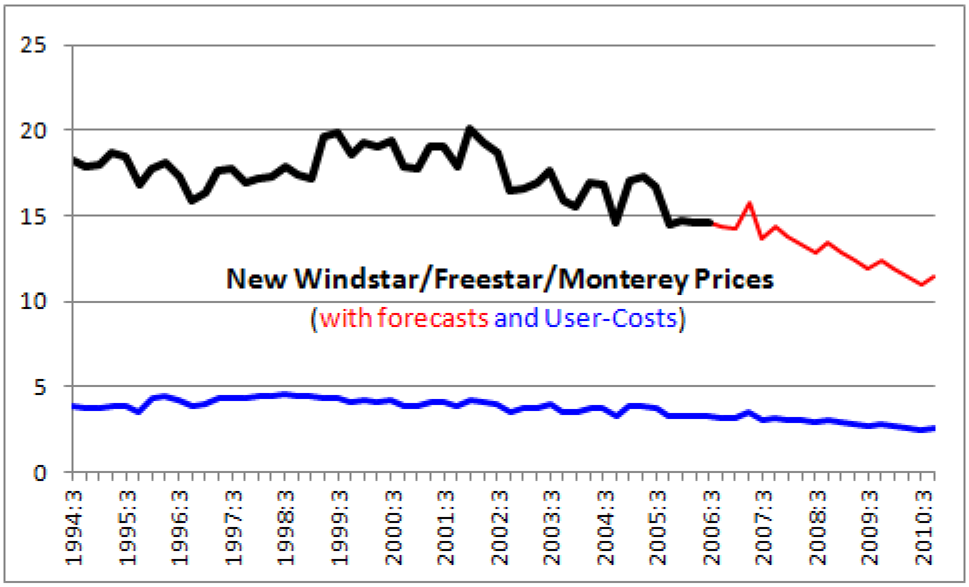
$$\beta_t = -14586.557 + 14.673986\,t - .0036902\,t^2 \qquad \hat{\sigma}_t = .045536 \qquad \text{(D.1)}$$

over 1994:III–2007:II and for frontier vintages

$$\beta_v = -161.93345 + .08057038\,t \qquad \hat{\sigma}_v = .0352862 \qquad \text{(D.2)}$$

over 1994:III–2006:III.[52] The trends offset each other for much of the period, such that the frontier-vintage Windstar price keeps in the range \$18,000 ± \$2,000, albeit with bumps. The corresponding per-year user-cost (discussed below) varies more smoothly about \$4,000. Figure D.1, below, plots the Windstar group's wholesale new price and user-cost. Finally, efforts to construct meaningful "stock" prices from the as-if-new transaction prices were fruitless, so I settled on booking capital stocks in the same prices that the quarter's investment and depreciation activities were conducted. Hence every period started with an update of its prices.

Figure D.1



An *Excel* workbook that carries out the accounting computations, "Windstar Capital Accounting.xlsx," accompanies this paper and is available upon request. The remainder of this appendix constitutes a user's guide to the workbook, which contains three tabs: "Individuals to Cohorts," "Cohorts to Wealth Stock," and "Cohorts to Productive Stock." The cell-references in what follows will only make sense with the workbook nearby.

The first 60 rows of the "Individuals to Cohorts" tab contain the names of specific Windstar/Freestar/ Monterey vintages or varieties, together with the preferred value of the efficiency-loss parameter for each, as well as how many vehicles were involved in estimating the variety's service-life density, as-if-new prices, and installations. The rightmost 100 columns (i.e., F through DA) contain positive numerical service-life densities for all the varieties, from age 3 out to topcode age 25. Selecting any variety name for focused consideration (into

[52] I used the numeric midpoint of each quarter—e.g., 1994:III = 1994.625, etc. Each vintage's β_v applied for the periods when it was the frontier—e.g., β_{95} applied for 1994:III-1995:IV. For nearly-right fitted values over the same interval, I took the within-interval average value—e.g. $\hat{\beta}_{95} = -161.93345 + .08057038 \times 1995.25$, since 1995.25 is the average of {1994.625, 1994.875, 1995.125, 1995.375, 1995.625, 1995.875}. Unlike the time-trend regression, which ran through 2007:II, the frontier-vintage regression concluded at 2006:III, after which I declared the '06 model-year had been supplanted by the '07 (despite the unsuitability of the latter for regression purposes). For vintages '99 and later, where separate variety coefficients are available, I formed the cross-variety vintage-average coefficient $\beta_{vintage}$ as the installation-weighted average of the variety-specific $\beta_{variety}$ coefficients. Fitted new prices are therefore: $\text{Exp}[c + \hat{\beta}_t + \hat{\beta}_{vintage} + \tfrac{1}{2}\hat{\sigma}_t^2 + \tfrac{1}{2}\hat{\sigma}_v^2]$, where the addition of half the estimated regression variances from (5.1) and (5.2) debiases the anti-LogNormal transformation.

cell A62) activates the worksheet's coding to compute the cohort-specific average resale-price profile Θ(s) (1.4), rental-price profile ρ(s) (A.1), and age-efficiency profile Φ(s) (A.2). All are set up as stationary functions of age, independent of calendar-time. And all three are "internal" only, lacking comparisons of their own as-if-new prices to that of the frontier vintage prevailing at a given calendar-date. Without such a comparison, we miss the half of depreciation that is attributable to obsolescence. Three big panels at the bottom of the tab (i.e., Θ(s) at B375:Y477, ρ(s) at Z375:AW477, and Φ(s) at AX375:BU477 — all before realized obsolescence) collect 24 columns each of the three cohort-level profiles — 23 for every vintage or variety where I fit cell-level individual profiles, plus overall averages.

The big panel of Θ(s) is copied to the rightmost reaches (IN5:PD156) of the "Cohorts to Wealth Stock" tab to await transformation to cohort-level resale-price profiles relative to the moving target of frontier-vintage prices. To accomplish this requires first assembling each variety's as-if-new prices and selecting the appropriate frontier-prices from among them, then multiplying the internal resale-price profiles by ratios of as-if-new- to frontier-prices. "Table of As-If-New Prices for Each Vintage/Variety through Every Quarter in the Data Where Each Could Have Been Present, with an Estimated Extension" (A2:AV60) collects the as-if-new prices, using the coefficient point-values from equation (3.1.2) that were printed in Table D.2 and which bound the table at column B and row 8. Frontier prices are collected at AV9:AV60. Trend regressions (D.1) and (D.2) are carried out at (AW5:BF60), so that future as-if-new and frontier prices may be estimated through 2031:II (A61:AV156), when the last of the '06's should sputter and die.

Multiplying the sales-price ratios to the internal resale-price profiles is the work of the big green-and-orange striped panel at (BU5:IK156). The horizontal orange stripes represent waves of realized obsolescence that tear through and substantially erode the remaining value of each established cohort. The resulting columns represent the worth of the old vans in terms of the (moveable) price of the frontier vintage.

The National Accounts punchline is at columns BI:BR. Investment in the form of 2,630,580 new vans across 49 quarters is collected at BP9:BP57. The net stock, in terms of units of the frontier vintage, is at BR9:BR155. The Windstar group peaks at 1,013,953 as-if-new units in 2001:4 and then inexorably declines. Consumption of vans — I'd call it depreciation, except that this is all phrased as new-equivalent van quantities — amounts to 2,630,580 units by the time it is complete at 2031:II. It is recorded at BQ10:BQ156.

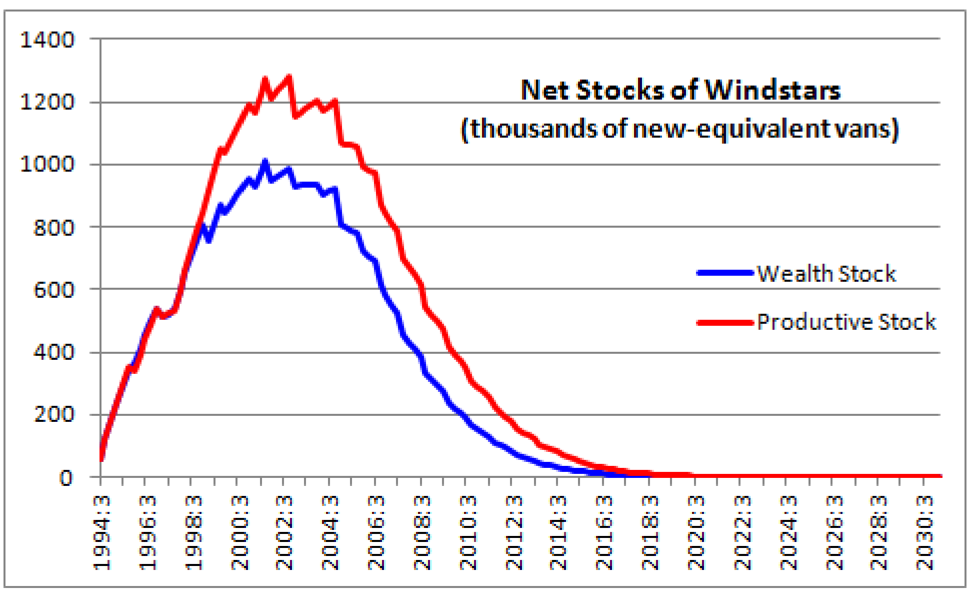
Columns BL, BM, and BN value investment, depreciation, and the net wealth stock in terms of the contemporary (frontier) investment price (column AV). The net stock peaks at $19.031 billion in 2002:I, on the strength of a 12.5 percent revaluation that quarter with the late arrival of the 2002 model year.

The third tab, "Cohorts to Productive Stock" is constructed by analogy to the second. Now, however, the tab's rightmost reaches (IN5:PD156) have the big ρ(s) panel from the first tab, representing variety-specific internal rental-price profiles. These also must be multiplied by ratios of as-if-new- to frontier- purchase prices to capture realized obsolescence. (Anticipated obsolescence is presumably built into *r*.) The construction and estimated extension of as-if-new prices is repeated at A2:AV156; only now, frontier prices (column AV) multiply the newest internal rental-price profiles (the ragged upper edge of the big ρ(s) panel) to form the user-cost series for the whole Windstar-Freestar-Monterey grouping at column BI.[53]

The third tab's big green-and-orange striped panel at (BU5:IK156) computes age-efficiency profiles for each vintage and variety in terms of the moveable overall user-cost, as the product of as-if-new prices and internal age-efficiency profiles, *divided by* the user-cost. Columns in this table tell how much an average asset (including retirees) from a vintage or variety would rent for at a given calendar-date, as a fraction of the rental price on the date's frontier vintage. This ratio of current rents is taken to equal the ratio of old-to-new marginal products in productivity analysis.

Columns BH:BR contain the summations that are useful to National Production Accounts. Investment is again 2,630,580 new vans at BP9:BP57, but the quantities represent the vans' immediately available new-rent equivalents, not the present discounted values of the full streams of their present and future rents. (That was the subject of the "Cohorts to Wealth Stock" tab.) The net productive stock is at column BR. It is substantially larger than the net wealth stock and peaks later, at 1,277,950 new-rent equivalent units in 2002:IV. Rental-quantity deterioration sums to 2,630,580 vans in column BQ. Plots of new-equivalent net wealth and net productive stocks, and of purchase-value depreciation and rental-value deterioration as fractions of the wealth and productive stocks, respectively, follow. The absence of constant decay rates, equal across wealth and productive stocks (which would characterize geometric decay), is obvious, and not only because new inputs fail after 2006 (forcing rates decay upward). The "crocodile teeth" highlight new model years' obsolescing power.

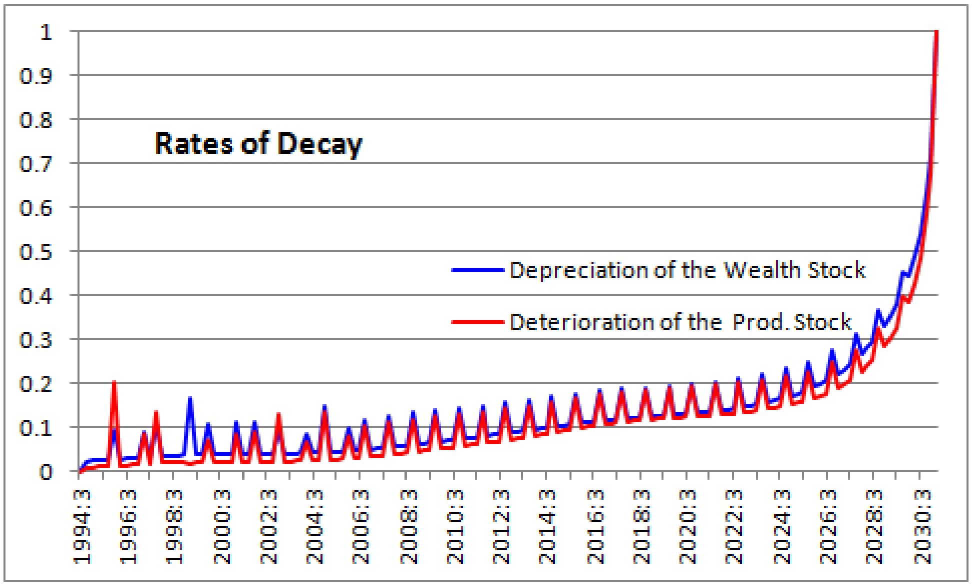
Figure D.2



What would be the impact on multifactor productivity measurement if all capital were captured as precisely as Windstars, and this even before quasifixity is considered?

[53] Except model-year '07.

Figure D.3



Movements in productive assets are priced by the user cost. Nominal values of investment, deterioration, and the productive stock are at columns BL:BN. Revaluations happen at the start of every quarter, in column BK.

Note the net productive stocks depend on the own rate of return, which itself depends on the expected, as opposed to realized, rate of change of a vintage/variety's own as-if-new price. So it is unlikely that the discounted sum of the entries of any column of BU5:IK156 (first multiplied through by Windstar-group user-cost, and then quartered to account for annual-level rental series at quarterly frequency) would agree with the frontier-price corresponding to the column. That offers a way to check on the accuracy of calculations. But it also forces National Accounts to split residual income into *expected* capital income and surprise profits or losses.