Implications of Consumer Heterogeneity on Price Measures for Technology Goods*

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Abstract

Using a new dataset on household purchases of personal computers (PCs), we document positive correlations between buyers' incomes and the prices they pay for seemingly identical PCs. These results suggest that firms may be successful at separating the market and charging different prices to consumers with different levels of willingness to pay. We consider the implications of this kind of market separation for price and quality measurement via a theoretical model based on Mussa and Rosen (1978). The model suggests that, in markets like these, standard methods that do not account for this heterogeneity can understate inflation in a cost-of-living context. Consistent with the model, our empirical work shows that controlling for income yields indexes that show slower price declines than seen in standard indexes. This understatement of the cost-of-living measure likely mitigates the unrelated upward biases found in recent studies by Bils (2009), Erickson and Pakes (2010), Broda and Weinstein (2010).

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1 Introduction

Prices for many goods tend to fall over the course of product cycles. Apparel, automobiles, and technological goods are examples of such products that are initially offered at high introductory prices and are then subsequently discounted until they are no longer available. There have been a host of explanations for such price declines including, but not limited to, fashion-type effects, process innovation, and intertemporal price discrimination. In this study, we analyze price declines for personal computers, a type of technology good.[1] We attribute the price declines to consumer heterogeneity over willingness to pay for quality and show that not accounting for this type of consumer heterogeneity can understate inflation growth in this sector.[2]

Technology goods are quite different from many other goods which show price declines over their product cycles. Unlike apparel, newly introduced technology goods are usually of higher quality than the goods they replace. Thus, any taste for the fashion (or newness) of a product may be tangled with the good's underlying better quality, as described by Bils (2009). Furthermore, technological goods are rapidly introduced into the market at staggered times. Accordingly, firms introduce newer, high-quality computers while still offering older, lower quality computers. This phenomenon does not occur in the automobile industry where new models are introduced only once per year and vintage is conspicuous.[3]

Our main finding is that standard methods that do not take this type of consumer heterogeneity into account can understate inflation for personal computers. There are few steps that we take to arrive at this result. First, using a new dataset on household purchases of personal computers (PCs) from Metafacts Inc., we document positive correlations between buyers' incomes and the prices they pay for identical PCs. High-income buyers therefore pay higher prices for identical PCs than lower-income buyers, suggesting

[1] See Berndt, Griliches, and Rappaport (1995) and Nelson, Tanguay, and Patterson (1994) for price measurement studies in this industry.

[2] This issue was first examined in Aizcorbe (2004) in a different context, where consumer heterogeneity of the type discussed here creates problems for the standard interpretation of price indexes as upper bounds to a true cost of living index.

[3] Fashion-type effects have been explored empirically by Pashigian (1988) for apparel and Corrado, Dunn, and Otoo (2006) for light vehicles.

that firms may be successful at separating the market and charging different prices to consumers with different levels of willingness-to-pay for quality. Thus, the data imply that richer households are not only buying more expensive, higher quality versions of products, as discussed in Bils and Klenow (2001), but they are also paying a premium for the relatively higher quality.

Second, we develop a dynamic theoretical model based on Mussa and Rosen (1978) which shows that this kind of market separation can lead to the downward sloping pricing patterns observed in the personal computer industry. The model includes two types of consumer heterogeneity: (1) heterogeneity over price elasticity and (2) heterogeneity over taste for newness. Thus, our model allows for two classes of consumer preferences; one is more general and the other is a more specific fashion-type effect. Firms can exploit both of these types of heterogeneity by selling higher quality products at a high markup to those willing to pay top dollar and selling lower quality products to those with lower willingness to pay. We then show conditions where such static price discrimination can exist in a dynamic equilibrium. This type of static separating equlibrium has been studied extensively in the industrial organization literature but, to our knowledge, has never been linked with dynamic pricing patterns in technological goods.[4] We believe that the assumptions needed for this type of market separation are plausible in a technology goods industry where newer high quality goods are sold contemporaneously with lower quality products that have been on the market for some time.

Third, we use the theoretical model to assess the implications of such consumer heterogeneity for the standard price indexes typically used to measure inflation. The model predicts that a "matched-model" price measure—which controls for quality by measuring the price change of the same computer over two-periods—will understate the true change in cost-of-loving when the price decline of that computer model is due to the firm separating the market between consumer types. The intuition for this result is that the matched-model price decline is not relevant to any single consumer type since the market is always separated in equilibrium. Consumers with high-willingness-to-pay

[4]These models of static price discrimination over vertically differentiated goods were developed by Mussa and Rosen (1978), Mirman and Sibley (1980), and Maskin and Riley (1984). Gowrisankaran and Rysman (2009) and Nair (2007) analyze a market where the distribution of consumers is changing over time such that firms are *intertemporally* price discriminating in dynamic equilibrium.

will always be enticed to buy the highest quality good available and consumers with low willingness to pay will always go for the cheaper option. Thus, the model posits that there is no gain in consumer surplus from a price decline of a single computer model as long as a better computer arrives the following period.

Finally, we consider several methods that an econometrician could use to assess how sensitive price indexes are to handling this effect due to consumer heterogeneity. Applying methods developed by Pakes (2003) to our household-level survey data, we compare hedonic price indexes that do not control for this heterogeneity with those that do by including income in the hedonic regression. Our results from the survey data show that the hedonic index that controls for income falls at about one tenth slower than a standard hedonic index. Furthermore, our results with the scanner data show that when we include the age of the PC in the hedonic price measure—a variable that we show is potentially correlated with consumer attributes— prices are *rising* over the sample period. Overall, the fact that these alternative segment-specific price indexes show flatter price declines is consistent with the theoretical prediction of the model. Specifically, our empirical result confirms that at least a portion of the price fall for a given computer model is due to consumer heterogeneity in taste which, we argue, ultimately should be held fixed in cost-of-living indexes.

It is important to note that our results contrast to those found in a few recent studies. Erickson and Pakes (2009) show that selection bias by exiting models will result in an overstatement of inflation in the matched-model index. Bils (2009) finds that forced substitutions by the BLS may be attributing too much growth to prices as opposed to quality, and Broda and Weinstein (2009) find that creation and destruction of products can hide quality upgrades that are not captured by indexes based on a fixed basket of goods. Our study is not refuting these results in any fashion, but rather illustrates an additional bias which should be taken into account when constructing cost-of-living measures.[5]

The study is organized as follows. In Section 2 we review the data. In Section

[5]There have studies other than ours that highlight a downward bias in BLS practices. Hobijn (2002) shows that inflation can be understated or overstated depending on the costs per quality unit between high and low cost products. Silver and Heravi (2005) show that inflation will be understated in the case of a matched-model index with sample degradation.

3, we review the model, both in the homogeneous one-product case, as well as the heterogeneous consumer two-product case. In Section 4, we show that the model is able to generate downward-sloping price contours where consumers with high willingness to pay pay a premium and purchase early in the product's life-cycle. In Section 5, we draw on the model's implications for price measurement. Here we show that the matched-model price measure matches the model's prediction of price inflation in the case of a homogeneous consumer but does not match the model's prediction in the heterogeneous case. In Section 6, we generate price indexes that control for consumer heterogeneity and demonstrate that prices are falling at a slower rate than the standard matched-model or hedonic price measures imply. In Section 7 we conclude.

2 Data

Our study uses data from two sources: a household survey data from the "Technology User Profile" (TUP) administered by MetaFacts and scanner data compiled by NPD Techworld. In the TUP data, we have access to four annual surveys that were conducted in 2001 through 2004. TUP is a detailed two-stage survey of Americans' use of information technology and consumer electronics products and services at home and in the workplace. The first stage is a screener, which asks for the characteristics of each head of household (such as income, education level, marital status, and presence of children). The second stage consists of the technology survey, which asks a multitude of questions ranging from brand, to year of purchase, to where the computer is used.[6] An observation in this data consists of household demographics and computer specifications including the price paid. We drop observations where we believe measurement error is present.[7] MetaFacts generates computer weights, which can be inferred as quantities on

[6] Although, the survey does include information on second and third computers owned by the household this study focuses on what the household reports as their "primary" computer.

[7] Specifically, we drop observations where the price paid is reported less than $100 and where the PC is younger than three years old at the time of the survey. We also isolate observations where the PC is used at home.

the household-computer type level.[8]

The NPD data are point-of-sale[9] scanner data sent to NPD Techworld via automatic feeds from their participating outlets on a weekly basis.[10] The data cover nine quarters, 2002:Q1 to 2004:Q3 and consist of sales occurring at outlet stores, thus manufacturers such as Dell that sell directly to the consumer are not included. Each observation consists of a model identification number, specifications for that model, the total units sold, and revenue. From units sold and revenue we calculate a unit price of each PC sold.[11] Figure 1 shows an example of prices for different models of 15-inch Hewlett Packard laptop computers over the sample period. This downward sloping pattern of pricing dynamics over the model's life cycle is ubiquitous across a range of brands and types of personal computers as can be seen in Table 1, which shows the average monthly price declines for the manufacturers in the NPD sample.

2.1 Demographics and Price

The pricing dynamics of personal computers seen in Figure 1 has potentially interesting implications for identifying demographic patterns in the data. Specifically, since there is variation in both the price and consumer types for a particular computer model, we can identify which types of consumers are purchasing at different points of the computer's time on the market. We analyze this variation in consumer types running the following fixed-effect regression in the TUP dataset:

[8] MetaFacts uses a two-stage algorithm in which they first create household weights from the U.S. Census data which are subsequently linked with computer data in the survey to create weights on the computer-household level.

[9] "Point-of-sale" means that any rebates or other discounts (coupons, for example) that occur at the cash register are included in the price reported; "mail in rebates" and other discounts that occur after the sale are not.

[10] The weekly data are organized into monthly data using the "Atkins Month Definition," where the first, second and third weeks of the quarter include four, four and five weeks, respectively.

[11] We remove observations where the PC was reported as "refurbished." We also remove observations where geographically isolated sales are likely to induce measurement error in our unit price variable. Specifically, we drop observations with less than 50 units sold in a month and less than 1000 units in the model's entire lifespan.

$$\ln P_{ij} = \alpha + \beta Z_i + \nu_j + \varepsilon_{i,j} \tag{1}$$

where Z_i represents a vector of demographic and location variables of consumer i while ν_j represents a fixed effect of computer model. Specifically, υ is a dummy variable representing year-RAM-speed-harddrive-form-manufacturer, where form represents whether the PC is a laptop or desktop of PC model j. As the TUP survey does not indicate the size of the monitor purchased, we limit our sample to desktop computers purchased with no monitor. Our results did not change using the entire sample. Since we are controlling for the type of computer j, a significant estimate of the coefficients on the demographic variables, β, indicates that consumers with different demographics paid different prices for the same computer. Given that the NPD scanner data indicates that most of the variation in price is declining from the good's introduction, in this study we assume that a positive coefficient on a demographic variable indicates that consumers of that type purchased earlier in the computer's life cycle.

Table 2 displays the results of a regression where we include income as a parametric variable (that is, lnIncome). We run a second regression where we include demographic variables other than income. In both specifications, income is positively and significantly correlated with price holding fixed the characteristics of the computer.[12] Specifically, the coefficient on lnIncome is approximately 0.09 and significant at the 3-percent level. This indicates that a ten percent fall in income would result in .9 percent lower price paid for any given computer.

3 Model

Our goal in this section is to formulate a model that can disentangle the distinct causes for the price declines seen in Figure 1, and simultaneously match the features

[12]For robustness purposes, we ran the same regression but included dummy variables indicating the income bin, instead of the logarithm of income. We also ran a pooled regression in which we included extra product characteristics which are subsequently included in the index analysis in the final section of the paper. The results do not change significantly for any of these regressions. We chose to display the fixed-effects regression for lnIncome for ease and clarity of composition.

of the data described in the preceding section—that higher income consumers purchase computers at higher price levels, presumably earlier in the computer's product life cycle. We describe a dynamic model that generates a stationary equilibrium such that a separating equilibrium occurs between consumers with different tastes for quality. The model generates parameters that govern the degree of consumer heterogeneity in taste for quality as well as parameters that govern the amount of price inflation and product quality growth. These parameters will later be used to generate an algebraic representation for a representative price contour that will be useful in decomposing price growth between consumer-specific, quality-specific, and inflation-specific factors.

We describe the model in a two-step fashion. First, we analyze the model under a setting of a representative consumer with a single-quality product firm. The representative consumer must decide between purchasing the product in the present period or waiting to purchase the following period when a higher quality product is introduced. The monopolist faces sufficiently low costs of adoption such that adoption occurs ever period. In equilibrium, the representative consumer purchases the product in the same period she enters the market.

In the second step, we extend the model by allowing for heterogeneity in consumer consumer taste. We describe the necessary and sufficient conditions of the parameters where the monopolist finds it optimal to separate the market between consumers of differing consumer tastes. The assumption of a monopoly is strong, but allows for tractable parameters of consumer preferences which facilitates our analysis in Section 5. Copeland and Shapiro (2010) analyze the single-product firm's pricing decision in an oligopolistic market. Interestingly, the end product of the single-product oligopolist and the multi-product monopolist who adopts every period (discussed in this study) is the same from the vantage point of the consumer. In both cases, consumers with differing tastes for quality are separated in equilibrium. That is, consumers with higher willingness to pay choose the high-quality product and pay a large markup while consumers with low willingness to pay choose the low-quality product and pay a low markup. Thus, our result of market seperation is not necessarily dependent on market structure.

3.1 Representative Consumer and Single-Quality Product Firm

We first look at the simple case of a representative consumer and a single-quality good firm. We solve for the dynamic demand function of the representative consumer in the setting where she takes as given that a new product of a different quality is offered in the subsequent period and that she can hold on to the good for an infinite period of time. One representative consumer enters the market each period, and we therefore abstract from the consumer's decision to upgrade her computer. We then describe the dynamic price path offered by the firm in equilibrium and show that this price path grows proportionally with the underlying exogenous growth rate of quality.

3.1.1 Demand

The representative consumer makes a discrete choice of whether to purchase a computer offered of quality x_t. As is standard in the industrial organization literature, we assume that the consumer's direct utility for the product is quasilinear with respect to the numeraire commodity, c_t. Specifically, the representative consumer solves the following maximization problem:

$$\max_{d \in \{0,1\}} d * f(x_t) + \tilde{\theta}_t c_t \tag{2}$$

subject to:

$$y_t = c_t + p(x_t)$$

where d is the consumer's choice of whether to purchase, $p(x_t)$ represents the price of a good with quality x_t,[13] $f(x_t)$ represents the total discounted utility of holding onto the good into the future,[14], y_t is income, and $\tilde{\theta}_t$ is the marginal rate of substitution between income and utility. The parameter $\tilde{\theta}_t$ is simply the relative utility received from holding the numeraire commodity as opposed to holding onto the product in period t. For example, if $\tilde{\theta}_t$ falls in value from period t to period $t+1$, then the value of the numeraire commodity falls relative to the good, and price inflation will ensue. As this is

[13]That is, price per quality is $p_t^x = \frac{p(x_t)}{x_t}$.

[14]Specifically, $f(x_t) = \frac{\nu(x_t)}{1-\beta}$ where $\nu(x_t)$ is the per period service flow from using the computer and β is the discount factor.

a time-varying parameter, one can think of this as a monetary shock where the value of the numeraire commodity fluctuates relative to the good. In Section 5, we will describe how $\tilde{\theta}_t$ fits into a welfare-based cost-of-living measure.

Substituting the budget constraint into the utility function we arrive at the standard indirect utility maximization problem of the consumer:

$$\max_{d\in\{0,1\}} d * (f(x_t) - \tilde{\theta}_t p(x_t)) + \tilde{\theta}_t y_t \tag{3}$$

As the outside good is the numeraire commodity, the consumer will purchase the good (that is, $d = 1$) if $p(x_t) \leq \theta_t f(x_t)$ where θ_t is the inverse of the time varying aggregate marginal rate of substitution $\tilde{\theta}_t$ (that is, $\theta = (1/\tilde{\theta})$).

We consider the representative consumer's two-period choice decision where a new representative consumer shows up to the retailer each period and subsequently faces the same two-period dynamic consumption decision. The consumer will decide to purchase a good in the current period t, or wait until next period $t+1$, when the firm offers a good with quality x_{t+1}. We assume that quality grows at constant rate σ^*, which is known to the consumer and the firm, but that $\tilde{\theta}_t$ follows the process $E[\tilde{\theta}_{t+1}|\Gamma_t] = \tilde{\theta}_t$ where Γ_t represents the consumer's information set at time t. Her consumption decision can be represented as:

$$\max\{f(x_t) - \tilde{\theta}_t p(x_t),\ \beta E[(f(x_{t+1}) - \tilde{\theta}_{t+1} p(x_{t+1}))|\Gamma_t]\}. \tag{4}$$

With $\beta > 0$, the consumer weighs the option of foregoing her purchase in period t in order to purchase a potentially higher quality product in period $t+1$. Therefore, after taking expectations, a consumer deciding between period t and period $t+1$ will purchase in period t if:

$$p(x_t) \leq \theta_t f(x_t) - \beta(\theta_t f(x_{t+1}) - E[p(x_{t+1})|\Gamma_t]). \tag{5}$$

Because growth in quality is fixed at $\frac{x_{t+1}}{x_t} = \sigma^*$, the consumer's valuation, $f(x_t)$ must grow at a rate that is a function of this exogenous technological growth. Thus, the consumer assumes that $f(x_t)$ grows at rate $\frac{\dot{f}}{f} = \sigma^f$ and assumes that $p(x_t)$ grows at an expected rate of $\frac{\dot{p}}{p} = E[\sigma^p|\Gamma_t]$.[15] The demand function of the representative consumer is then:

$$d(p(x_t)) = \begin{cases} 1 & \text{if} \quad p(x_t) \leq \delta\theta_t f(x_t) \\ 0 & \text{if} \quad p(x_t) > \delta\theta_t f(x_t) \end{cases} \tag{6}$$

where $\delta = \frac{1-\beta(1+\sigma^f)}{1-\beta(1+E[\sigma^p|\Gamma_t])}$.

3.1.2 Equilibrium

The firm is a forward-looking monopolist who takes the growth in (upstream) quality as exogenous and faces zero adoption costs. It sets a price each period of product quality x_t given the consumer's dynamic demand function, equation (6). That is, the firm maximizes its profits $p(x_t) - C(x_t)$, where $C(x_t)$ is the cost of supplying good x_t. Thus, we abstract from the firm's dynamic adoption decision and assume that marginal and adoption costs are sufficiently low that the firm will choose to adopt a new product every period.

The firm's profit maximization problem in the static setting is quite trivial. It charges the highest price it can and reaps all of the consumer surplus of the representative consumer each period by setting $p(x_t) = \theta_t f(x_t)$. In the dynamic setting, however, the firm must set a path of future prices such that the market clears each period. It turns out the equilibrium path of prices is such that prices grow proportionally with the growth rate of utility. This result is provided in the following theorem:

Theorem 1. *The equilibrium price path is $\sigma^p = \sigma^f$ such that $\delta = 1$.*

Proof. See Appendix $\square$

Theorem 1 tells us that in equilibrium, prices will grow at the same rate as the growth in utility, σ^f. It follows that the firm sets $\delta = 1$ and markets clear each period at the price $p(x_t) = \theta_t f(x_t)$. As it is always optimal for the firm to set prices that grow with the consumer's valuation of the good—which is dependent on the good's quality—

[15] That is, $f(x_{t+1}) = (1 + \sigma^f)f(x_t)$ and $E[p(x_{t+1})|\Gamma_t] = (1 + E[\sigma^p|\Gamma_t])p(x_t)$.

waiting until the next period to purchase the better-quality good will yield zero consumer surplus regardless of the representative consumer's discount factor. Thus, in equilibrium, the representative consumer who enters the market in period t purchases the product in in the same period.

3.2 Heterogeneous Consumers and Multi-Quality Product Firm

We extend the model by adding heterogeneity in consumer valuations over quality, as well as adding heterogeneity in quality of the underlying product. Since $\tilde{\theta}_t$ is common to all consumers, and consumers perceive that $E[\tilde{\theta}_{t+1}|\Gamma_t] = \tilde{\theta}_t$, we can normalize $\tilde{\theta}_t = 1$ without any loss in generality.[16] Heterogeneity in quality is described via the parameter r, which identifies the "rank" of the quality of the good relative to other goods at time period t. It follows that for any two goods, $x_{j,t} > x_{k,t}$ if $j < k$. An important assumption we make in order to keep the model tractable is that the number of goods entering each period is equal to the number of goods exiting. This assumption implies that, each period, two different goods are offered, one with rank 1 and one with rank 2. It follows that any good with rank r grows in quality at rate σ^* between periods t and $t+1$.

There exist two types of consumers, $i \in \{H, L\}$, where a proportion α of consumers are H-type consumers and the remaining proportion, $1-\alpha$, are L-type consumers. Consumer i's indirect utility maximization problem can then be defined as:

$$\max_{d^i \in \{0,1\}} d^i * (b^{i,r} f(x_{r,t}) - p(x_{r,t})) + y_t^i \tag{7}$$

where b represents a consumer-specific valuation attached to the discounted service flow such that $b^{i,r} f(x_{r,t})$ is the value that a consumer of type i places on good with quality $x_{r,t}$.[17] In order to keep the model open to differing types of taste distributions, we analyze two cases of heterogeneity over consumer tastes:

[16] The importance of movements $\tilde{\theta}_t$ will become apparent in Section 5.

[17] As shown in Tirole (1988), one can reinterpret this preference parameter as the inverse of the marginal rate of substitution between income and quality by dividing through by $b^{i,r}$ to give the utility as: $f(x_t) - \frac{p(x_{t,r})}{b^{i,r}}$.

- *Case 1*: Heterogeneity in Price Elasticity: $b^H > b^L$ for any r
- *Case 2*: Heterogeneity in Taste for Rank (or Newness): $b^{H,1} > b^{L,1}$ and $b^{H,2} < b^{L,2}$

Case 1 is the scenario where the H-type consumer is less price elastic than the L-type consumer and therefore has a higher willingness to pay for any given quality. Case 2 is the scenario where the H-type consumer places a strong aversion towards relatively low-quality goods. For instance, case 2 would be a situation in which H-types are gamers and need the latest technology products to keep pace with the latest software. It could also be a situation where H-type consumers have a high cost for searching for information about products (that is, they simply ask the retailer for the highest quality product when they walk through the door).

3.2.1 The Static Case

The firm's problem in the two-consumer-type static case is to maximize profits given consumer utility $b^{i,r}f(x_{r,t})-p(x_{r,t})+y_t^i$ if $d^i = 1$. This is analogous to the static case of the representative consumer, however now the firm has the option to separate the market by offering different quality products in order to increase its profits. We look at the specific case where the firm can choose to offer the good in two different qualities, where good 1 represents the high-quality good and good 2 represents the low-quality good in any period t. Thus, we are splitting x_t of the preceding section into a two-dimensional vector of high- and low-quality goods $\{x_{1,t}, x_{2,t}\}$. For the separating equilibrium to occur, the firm must be able to separate the market and also find it profit-maximizing to do so. Such prices are delineated by Mussa and Rosen (1978):

- *Case 1*: The firm's prices must satisfy: $p(x_{1,t}) = b^H f(x_{1,t}) - (b^H - b^L)f(x_{2,t})$ and $p(x_{2,t}) = b^L f(x_{2,t})$.
- *Case 2*: The firm's prices must satisfy: $p(x_{1,t}) = b^{H,1} f(x_{1,t})$ and $p(x_{2,t}) = b^{L,2} f(x_{2,t})$.

Under case 2, the firm will separate the market by offering each consumer his reservation price: $p(x_{r,t}) = b^{i,r} f(x_{r,t})$. Under case 1, however, the H-type values the rank-2 good more than the L-type consumer, and the firm must lower the price of the rank-1 good to dissuade the H-type from deviating and purchasing the rank-2 good. Specifically, the bound is lowered by the extra utility the H-type would have received over the L-type of consuming the rank-2 good, $(b^H - b^L)f(x_{2,t})$. This lowering of the price ensures that the H-type consumer does not purchase the rank-2 good instead of the rank-1 good (that is, that ensure the equilibrium is incentive compatible). In either case, the firm will find it profit-maximizing to separate the market as long as α is sufficiently small—that is, the number of L-types is sufficiently large (see Tirole (1988)).

3.2.2 Stationary Equilibrium in a Dynamic Setting

The consumers' dynamic problem in the multi-quality product case is similar to the consumer's problem in the single-quality case, however, there is now the added option of purchasing a different rank good in the following period. Thus, for markets to clear each period, we must find conditions for which it is not optimal for the consumer to wait until the following period to purchase a different rank good.

It can be trivially shown that it is never optimal for the L-type consumer to wait to purchase the rank-1 good in the following period in either case 1 or case 2. It is also never optimal for the H-type to wait to purchase the rank-2 good in case 2.[18] In case 1, however, this decision is dependent on the size of the discount factor. Specifically, it is not optimal to wait to purchase the rank-2 good only if the discount factor, β lies below $\frac{1}{1+\sigma^f}$ (that is, $\beta < \frac{1}{1+\sigma^f}$).[19] Thus, if the H-type is sufficiently impatient, she will choose not to wait to purchase the rank-2 good in the subsequent period. In the setting where the consumer is choosing only between the same rank good in the current and subsequent periods, the demand decision is analogous to the representative consumer, one-quality good model:

$$\max\{b^{i,r}f(x_t) - p(x_t),\ \beta E[(b^{i,r}f(x_{t+1}) - p(x_{t+1}))|\Gamma_t]\} \tag{8}$$

[18] In this case, the firm prices at $p(x_{1,t}) = b^{H,1}f(x_{1,t})$ and $p(x_{2,t}) = b^{L,2}\theta_t f(x_{2,t})$. It holds that $b^{H,2}f(x_{2,t+1}) < p(x_{2,t+1})$ and it is never optimal for the H-type to wait.

[19] This is equivalent to the condition $\sigma^f < \frac{1-\beta}{\beta}$ described in the proof of Theorem 1. This restriction is found by setting $b^H f(x_{1,t}) - p(x_{1,t}) > \beta b^H f(x_{2,t+1}) - p(x_{2,t+1})$ under the conditions that $p(x_{1,t}) = b^H f(x_{1,t}) - (b^H - b^L)f(x_{2,t})$, $f(x_{2,t+1}) = (1+\sigma^f)f(x_{2,t})$, and $p(x_{2,t+1}) = b^L f(x_{2,t+1})$.

where $b^{i,r}$ represents the consumer-specific utility for consuming a product with quality $x_{r,t}$. The demand function of consumer i for a good with rank r is then:

$$d^i(p(x_{r,t})) = \begin{cases} 1 & \text{if} \quad p(x_{r,t}) \leq \delta b^{i,r} f(x_{r,t}) \\ 0 & \text{if} \quad p(x_{r,t}) > \delta b^{i,r} f(x_{r,t}) \end{cases} \tag{9}$$

Each period, the firm separates the market between L-type and H-type consumers and a separating equilibrium ensues. It can be shown that Theorem 1 holds and the firm's optimal price path is analogous to the case of the representative consumer.

When the firm is able to separate the market it will offer two goods each period and charge a premium on rank-1 goods sold to H-type consumers. The larger the degree of consumer heterogeneity, the more the firm can charge to the H-type relative to the low-type consumer $\frac{p(x_{1,t})-p(x_{2,t})}{p(x_{2,t})}$, and the higher the premium, Δ, the firm can charge on the highest quality good. Substituting the market-clearing prices into this definition of the premium, we can represent it as a function of the underlying taste and quality parameters:

- *Case 1:* $\Delta = \frac{b^H}{b^L}\left[\frac{f(x_{1,t})}{f(x_{2,t})} - 1\right]$
- *Case 2:* $\Delta = \frac{b^{H,1} f(x_{1,t})}{b^{L,2} f(x_{2,t})} - 1$

Thus, the premium will increase with $\frac{b^H}{b^L}$ and $\frac{f(x_{1,t})}{f(x_{2,t})}$ and will be independent of time as long as the b's are constant.

4 Generating Price Contours

We now show that the model generates price contours that resemble those seen in both the NPD data and TUP data. That is, we show that the model is able generate downward sloping price contours where H-types (that is, high income consumers) purchase newly introduced rank-1 goods and low-types (that is, low-income consumers) purchase rank-2 goods.

We introduce a new variable indicating a computer model, where computer model j lives for two periods, and with introduction of a new, higher quality model every period, a given model will fall in rank, from rank-1 to rank-2, between the first and second periods. In the third period, we assume the model exits the market place. Thus, the former highest quality product (that is, rank-1), becomes the second highest quality good (that is, rank-2) in the subsequent period but the model number does not change over periods. We label the price path generated over time of any given computer model a "price contour," which is analogous to the price contours of computer models observed in the data.[20]

Sequential models are labeled alphabetically, such that model b replaces model a as the rank-1 good in period t, and subsequently in period $t+1$, model c replaces model b as the rank-1 good. Since prices of the entire fleet of goods grows at rate σ^f, it follows that $p(x^b_{1,t+1}) = (1+\sigma^f)p(x^a_{1,t})$, and we can show the price contour as the relationship between prices in any period for a given computer model:

$$\begin{aligned} p(x^b_{1,t}) &= (1+\Delta)p(x^a_{2,t}) \\ &= \left(\frac{1+\Delta}{1+\sigma^f}\right)p(x^b_{2,t+1}) \end{aligned} \tag{10}$$

which indicates that in the case of two different quality goods offered each period, price contours will be downward sloping if $\Delta > \sigma^f$, flat if $\Delta = \sigma^f$, and upward sloping if $\Delta < \sigma^f$. Substituting the definition of Δ into (10), and using the fact that the firm that adopts the new model will choose a model at the technological frontier (that is, $\frac{f(x_{1,t})}{f(x_{2,t})} = \sigma^f + 1$) it follows that the price contour will always be downward sloping if $b^H > b^L$. Figure 2 depicts the price contours generated under the model when θ is fixed to equal 1. The upper dotted line indicates the fixed growth path of prices for the rank-1 good, and the lower dotted line for the rank-2 good. Both price paths are increasing due to positive quality growth, and thus each of these paths is growing at rate $1+\sigma^f$. The figure demonstrates that a larger premium will widen the distance between the two price paths and therefore push the price of the rank-2 good down relative to the rank-1 good between time periods. The larger is the premium, the larger the price fall of a given computer model must be in order to compensate the premium within the two growth

[20] Price contours will be downward sloping if $p(x^j_{1,t}) > p(x^j_{2,t+1})$, flat if $p(x^j_{1,t}) = p(x^j_{2,t+1})$, and upward sloping if $p(x^j_{1,t}) < p(x^j_{2,t+1})$.

paths. Thus, the model posits that the price for any given computer model is being driven down from period t to period $t+1$ due to a drop in the premium charged to the H-type consumer over the L-type consumer.

This idea can be more easily intuited under case 2 and representing the ratio $\frac{b^{H,1}}{b^{L,2}}$ as:

$$\gamma + 1 = \frac{b^{H,1}}{b^{L,2}}, \tag{11}$$

which allows us to represent the premium as

$$\begin{aligned} \Delta &= (\gamma+1)(\sigma^f+1) - 1 \\ &= \gamma + \sigma^f + \sigma^f\gamma. \end{aligned} \tag{12}$$

This relationship above shows that a significant portion of the premium between rank-1 and rank-2 goods is owed to heterogeneity in taste for quality. To relate this idea with the patterns seen in the data, we depict equation (12) in Figure 3 which shows the pricing dynamics of two models (a and b) over the course of three time periods, ($t-1$, t, and $t+1$). The figure shows the premium decomposed between γ, the portion due solely to the firm exploiting $b^{H,1} > b^{L,2}$ by separating the market, and $\sigma^f + \sigma^f\gamma$, the portion of the premium due to utility growth stemming from growth in the underlying quality of the distribution, σ^*. The figure demonstrates that the sole source of the price fall of good a is due to $\gamma > 0$. When heterogeneity of relative value over rank diminishes, (that is, when $b^{H,1} = b^{L,2}$) such that $\gamma = 0$, price contours become flat. As long as $\gamma > 0$, price contours will be downward sloping due to the fact that firms charge a high price on rank-1 goods in order to capture their consumer surplus.[21]

An important thing to note here is that the downward sloping nature of the price contours has nothing to do with the durability of the good. That is, the model does not posit that the distribution of consumers is changing over the life of the product whereby high-willingness-to-pay consumers drop out of the market after purchasing early in the

[21] Since the downward price fall is owed to $\gamma > 0$, the model shows downward sloping price contours is not related to the good changing in physical quality between periods. This can be seen by looking at the path of $f(x)$, which stays constant between periods for any specific product, x^j. This is inherent in our assumption that $\frac{f(x^b_{1,t})}{f(x^a_{2,t})} = 1 + \sigma^f$, which means that for $f(x^b_{1,t}) = (1+\sigma^f)f(x^a_{2,t}) = f(x^b_{2,t+1})$.

product cycle.[22] Rather, prices are falling due to two conditions: (1) firms are price discriminating in the static sense, and (2) higher-quality goods are being offered in the subsequent period. Thus, the combination of market separation and technology growth is essential to this outcome of intertemporal price declines.

5 Implications for Price Measurement

Having set up the model and showing its link with the data we are now in a position to analyze the implications of market separation on price-change measures. First, we show that price changes, or "price relatives," are a component of most cost-of-living indexes. Specifically, cost-of-living indexes can be represented by a weighted sum of price relatives between any two price regimes. A matched-model index is simply a weighted sum of price relatives between individual model numbers. We then show that the extent to which the matched-model measure differs from actual inflation (that is, movement in θ_t) will depend on some key parameters of the model described above. In particular, if the two measures of inflation are to overlap, one must control for the component of the premium due to market separation. Finally, we show that one method of controlling for discrepancy is through hedonic techniques.

5.1 Cost-of-Living Indexes

5.1.1 Matched-Model Cost-of-Living Index

A cost-of-living index tracks the compensation needed to keep a specific consumer's utility fixed when transferring to a new price regime. The consumer's utility is subsequently based on the consumer's preference ordering of goods. Let R_i denote the preference ordering of consumer type i between consumption bundles Q and Q', such that $Q \; R \; Q'$ if and only if $U^i(Q) \geq U^i(Q')$, where $U^i(\cdot)$ is the direct utility function (see Pollak (1989)). Thus, consumer i's particular direct utility function, $U^i(\cdot)$, depends on how she orders goods according to her individual preference, R^i. Consumer i's compensated

[22]See Stokey (1979), Conlisk, Gerstner, and Sobel (1984), and Nair (2007) for an examination of this type of "intertemporal price discrimination."

demand function (that is, Hicksian demand correspondence), $H_i(P, s|U^i)$, gives the level of demand that would arise if she were compensated for any change in price, p, to keep her utility fixed at level s. Finally, we let $W(P, s|U^i)$ represent the minimum expenditure required to attain a particular utility level, s, such that $W(P, s|U^i) = P \cdot H(P, s|U^i)$.

A cost-of-living index captures the change in the minimum expenditure needed to keep consumer i's utility level fixed at level s if the price vector is varied. Specifically, it is the ratio of the minimum expenditures, W, under two different aggregate price states, P and $\tilde{P}$, such that:

$$I(P, \tilde{P}, s, U^i) = \frac{W(\tilde{P}, s|U^i)}{W(P, s|U^i)} = \frac{\tilde{P} \cdot H(\tilde{P}, s|U^i)}{P \cdot H(P, s|U^i)}, \tag{13}$$

where the denominator represents the "reference" period and the numerator the "comparison" period. This representation of the cost-of-living index highlights the fact that in comparing the minimum expenditure required to obtain utility s, we must hold fixed consumer i's particular direct utility function. Thus, the index is designed to compare the minimum expenditures of a particular type of consumer with preference ordering R_i.

Under the simplified scenario of a representative consumer, whose preferences are fixed, in a market with no entering or exiting goods, a Laspeyres price index can be calculated as an approximation to $I(P, \tilde{P}, s, U^i)$ where the quantities purchased in the reference period are used as a substitute for the Hicksian demand:

$$I^L(P, \tilde{P}) = \frac{\tilde{P} \cdot Q(P)}{P \cdot Q(P)} = \frac{\sum q^j \tilde{p}^j}{\sum q^j p^j} = \sum \omega^j \frac{\tilde{p}^j}{p^j}, \tag{14}$$

where q^j represents the quantity of good j, ω^j is the expenditure share of good j in the reference period, and $\frac{\tilde{p}^j}{p^j}$ is the "price relative" between different price regimes for good j.[23] The Laspeyres index represents an *upper* bound on $I(P, \tilde{P}, s, U^i)$ because it does not take into account the fact that the consumer can substitute to cheaper goods when one moves to the comparison price regime. Thus, the Laspeyres assumes that the representative consumer's utility level, s, is held fixed by keeping the quantities of goods fixed.

[23] The Paasche index is analogous to 14 except the quantities in the comparison period are held fixed.

In the context of creating a cost-of-living index for goods whose quality is distinguished by its model number, (14) can be interpreted as a matched-model price index. Specifically, model numbers are differentiated by j and the base and reference periods represent two different time periods. Thus, a Laspeyres matched-model price index is a weighted sum of price relatives and represents an upper bound on the amount of dollars needed to keep utility fixed when prices of existing models change between periods t and $t+1$.

5.1.2 The Hedonic Cost-of-Living Index

Hedonic price indexes are primarily used in cases when a specific good exits or enters the market and a comparison or reference period price cannot be calculated under specification (14). In such a scenario, Pakes (2003) shows that an expectation of the price of the good, conditional on its observable characteristics, can be included as a substitute for the missing observable price. This expectation of the price is based on the hedonic function. Specifically, for good j with observable characteristics x^j the hedonic function would be represented by:

$$h(x^j) = E[p^j|x^j]. \tag{15}$$

The hedonic function is the predicted value of the price of good j conditional on its vector of characteristics, x^j. Thus, the hedonic assumes that the representative consumer's preference ordering of goods, R, can be decomposed into the characteristics of the underlying good, x^j. This procedure theoretically allows the prices of exiting goods to be compared to their expected prices in the subsequent period; similarly entering goods can be compared to their expected price in the previous period.

Letting C_t represent the choice set available in period t, we can create a hedonic function, $h_t(x)$, for each time period in the sample. A Laspeyeres index (or any other specified weights) can then be generated in terms of the predicted prices from the hedonic functions:

$$I^H(P_t, P_{t+1}) = \frac{\sum_{j \in C_t} h_{t+1}(x^j) q_t^j}{\sum_{j \in C_t} h_t(x^j) q_t^j} = \sum_{j \in C_t} \omega^j \frac{h_{t+1}(x^j)}{h_t(x^j)}, \tag{16}$$

where $h_t(\cdot)$ is the tth period predicted value of the hedonic regression, ω^j is the expenditure weight of good j, $\frac{h_t(x^j)q_t^j}{\sum_{j\in C_t} h_t(x^j)q_t^j}$, and C_t represents the basket of goods available in period t.

Under certain conditions, the hedonic price relative will overlap with that of the matched-model. This can be see in Figure 4, where we convert the depiction of price versus time into a relationship between price and quality of the good. In this particular example, the hedonic is perfectly identified by the good's model number, the fall in the price relative of the hedonic index is equivalent to that of the matched-model—that is, $\frac{h_{t+1}}{h_t} = \frac{p_{t+1}(x_2^j)}{p_t(x_1^j)}$.

5.2 Price Measures in the Context of the Model

We now show that the model delineated in Section 3 allows us to calculate the compensating variation between any two price regimes. Thus, we can can compute the amount of dollars needed to keep a given consumer's utility fixed between any two price regimes and then compare the matched-model and hedonic price measures to the measure of inflation generated from the model. We show that the the degree of consumer heterogeneity plays a large role in how our model and the matched-model price measures differ.

5.2.1 Homogenous Consumers

We start with the representation of the matched-model in the homogenous consumer case where the firm sets a price $p(x_t) = \theta_t f(x_t)$. The benefit of the model is that we can differentiate between aggregate movements in θ_t and growth in quality x_t. This lets us see hypothetical scenarios of what prices would look like under different price regimes. The linkage between the parameter θ and the cost-of-living index described above becomes apparent by reviewing the consumer's direct utility function, equation (2). The intuition is most straightforward in the case when the consumer is only consuming the numeraire commodity. It follows that if $\tilde{\theta}$ falls from value, say $\tilde{\theta}_t = 1$ to $\tilde{\theta}_{t+1} = 0.25$, the consumer needs $\frac{\tilde{\theta}_t}{\tilde{\theta}_{t+1}} = 4$ times the amount of the numeraire commodity for his utility to be kept fixed. Thus, movements in $\tilde{\theta}$ capture the amount of expenditure needed to keep the consumer's direct utility fixed between different aggregate regimes—just as the cost-of-living index is intended to do.

The same intuition follows in the case when the consumer decides to purchase the product (i.e., $d = 1$.). In this case, movements in $\tilde{\theta}$ induce movements in the price charged to the consumer by the firm. With some added notation, it follows from the model that under the regime θ_a (where $\theta_a = 1/\tilde{\theta}_a$), the firm will set a price:

$$p_a(x_t) = \theta_a f(x_t). \tag{17}$$

Thus, the firm will price a good of quality x_t at $p_{t+1}(x_t) = \theta_{t+1} f(x_t)$ under regime θ_{t+1} and will price a good of quality x_{t+1} at $p_t(x_{t+1}) = \theta_t f(x_{t+1})$ under regime θ_t. As in Aizcorbe (2005), we can decompose *total* price growth (TPG) between that due to quality and that due to prices caused by unexpected movements in θ_t.

$$\text{TPG} = \frac{p_{t+1}(x_{t+1})}{p_t(x_t)} = \left[\frac{p_{t+1}(x_{t+1})}{p_{t+1}(x_t)}\right]\left[\frac{p_{t+1}(x_t)}{p_t(x_t)}\right]. \tag{18}$$

In the simple case where $f(x) = x$, it follows that $\text{TPG} = [1+\sigma^*][\frac{\theta_{t+1}}{\theta_t}]$.[24] If we want the cost-of-living index to measure the amount of the numeraire commodity (e.g. dollars) needed to keep the consumer's utility fixed between aggregate price regime changes, then the matched-model index needs to be a good measure of $\frac{\theta_{t+1}}{\theta_t}$. With homogenous consumers, the matched-model price relative does a good job of measuring this:

$$\text{MM} = \frac{p_{t+1}(x^j)}{p_t(x^j)} = \left[\frac{p_{t+1}(x^j)}{p_{t+1}(x^j)}\right]\left[\frac{p_{t+1}(x^j)}{p_t(x^j)}\right] = \frac{\theta_{t+1} f(x^j)}{\theta_t f(x^j)}, \tag{19}$$

which means $\text{MM} = \frac{\theta_{t+1}}{\theta_t}$, and the matched model is picking up exactly what it is supposed to—"inflation" as measured by a change in price regimes. Likewise quality growth would be measured as the deflated portion of (18) using our matched-model measure of inflation, which results in $1 + \sigma^* = 1 + \sigma^f$ which again is accurate since we assumed in this case that $f(x) = x$.

[24] If we wish to allow for curvature of the utility $f(\cdot)$ such that $f(x) = x^c$ for some constant $|c| < 1$ it follows that $\text{TPG} = [(1+\sigma^*)^c][\frac{\theta_{t+1}}{\theta_t}]$.

5.2.2 Heterogenous Consumers

We now show that the matched-model price relative performs poorly in the case of heterogeneous consumers. Our hypothetical prices found in the heterogeneous case can thus be varied in the same way as in the homogenous case. For example, in case 2, $p_{t+1}(x_{1,t}) = \theta_{t+1} b^{H,1} f(x_{1,t})$ is the price charged for good $x_{1,t}$ under regime θ_{t+1} and $p_t(x_{1,t+1}) = \theta_t b^{H,1} f(x_{1,t+1})$ is the price charged for good $x_{1,t}$ under regime θ_t. Decomposing the matched-model price relative in terms of quality and prices then results in:

$$\mathrm{MM} = \frac{p_{t+1}(x^j_{2,t+1})}{p_t(x^j_{1,t})} = \left[\frac{p_{t+1}(x^j_{2,t+1})}{p_{t+1}(x^j_{1,t})}\right]\left[\frac{p_{t+1}(x^j_{1,t})}{p_t(x^j_{1,t})}\right]. \tag{20}$$

Plugging in the parameters from the model results in:

- *Case 1*: $\mathrm{MM} = \left[\frac{b^L + b^L\sigma}{b^L + b^H\sigma}\right]\left[\frac{\theta_{t+1}}{\theta_t}\right]$
- *Case 2*: $\mathrm{MM} = \left[\frac{b^{L,2}}{b^{H,1}}\right]\left[\frac{\theta_{t+1}}{\theta_t}\right]$.

The above relationship shows that the matched-model price relative is mis-measuring "inflation," $\frac{\theta_{t+1}}{\theta_t}$, due to the fact that consumers have differing tastes, (that is, $b^H > b^L$).[25] Specifically, the matched-model price measure is formulated under the hypothesis that an H-type consumer would switch her preference from a rank-1 good to a rank-2 good whereas the model is constructed under the separating equilibrium where this does not occur. In other words, the model is constructed such that, not only does the H-type consumer not wish to wait to purchase the same computer in the subsequent period, but even if she did wait, given her subsequent choice of products and prices she would be enticed to purchase the higher quality good. Thus, the relevant price changes for this type of consumer are those prices of computers with the highest quality.

Overall, the extent to which the matched-model measure of price inflation falls below the actual price inflation implied by the model will be a function of the degree of consumer heterogeneity in taste. A larger degree of heterogeneity will imply a larger markup charged to the H-type consumer, a larger markup drop in the subsequent period, and therefore a larger price fall measured by the matched-model index.

[25] Note that under case 2, this condition means that $\mathrm{MM} = \left[\frac{1}{1+\gamma}\right][1+\sigma^\theta]$, where $\frac{1}{1+\gamma} = \frac{1+\sigma}{1+\Delta}$, which is the same condition as (10).

5.3 A Segment-Hedonic Cost-of-Living Index

The relationship in (20) reveals if the econometrician is trying to match the price measure implied by the model, she would ideally like to control for the premium generated by consumer heterogeneity in calculating the price relatives. A measure of price movements that control for the specific type of consumer could be constructed by looking at the price paid by a specific type of individual—either H-type or L-type. For instance, one could generate price relatives based on comparing the price of the same PC model j with the same rank but under different price regimes:

$$\frac{h_{t+1}(x_{1,t}^j)}{h_t(x_{1,t}^j)} = \frac{\theta_{t+1} b^{H,1} f(x_{1,t}^j)}{\theta_t b^{H,1} f(x_{1,t}^j)} = \frac{\theta_{t+1}}{\theta_t}. \tag{21}$$

With a hedonic regression one would want to impute the price that the H-type will pay for a given computer model and a *given choice set* (that is, if it were a rank-1 good), *but* she happened to live in period $t+1$ instead of period t, which corresponds to the numerator of equation (21), $h_{t+1}(x_{1,t}^j)$. Thus, this price change measure is capturing the experiment of dropping an individual off on one island with regime θ_t and then dropping her off on another island with regime θ_{t+1} and then comparing the amount of dollars needed to keep her equally happy on both islands, given that the exact same computers were available on both. An index of this sort would be a segment-hedonic price measure since the hedonic measure would control for the characteristics of the consumer and hence the markup charged to this type of individual.[26]

With enough consumer specific price data, one can directly compute the imputed price $h_{t+1}(x_{1,t}^j)$ above by isolating consumers into specific consumer type bins, i. This exercise requires obtaining a well defined consumer attribute variable which distinguishes between "consumer types," as well as a large enough data set such that one could find at least two individuals of the same consumer type, i, purchasing the same product between any two periods. When the size of the data is limited, however, one could

[26]The index relevant to a consumer of segment i would be $I(P^i, \tilde{P}^i, s, U^i) = \frac{\tilde{P}^i \cdot H(\tilde{P}^i, s|U^i)}{P^i \cdot H(P^i, s|U^i)}$, which shows that different consumer segments will have different cost-of-living indexes not only because they have different preference ordering (that is, different U^i's), but also because they pay different prices for the same bundle of observable characteristics as described in the model.

generate predicted prices for a specific vector of good and consumer characteristics using the hedonic function described above. In this case, the econometrician needs to know either the consumer-specific attributes which differentiate consumers by consumer type, or she needs to observe the characteristic of the good that the firm uses to separate the market.

If the econometrician knows the specific characteristic the firm uses to separate the market, then this variable can be included in the hedonic regression to control for the distinct prices charged to each type of consumer:

$$h^r(v^{i,j}, x^j) = E[p^{i,j}|v^{i,j}, x^j], \tag{22}$$

where $v^{i,j}$ is the characteristic of good j on which type i consumers place a specific value. If rank is highly correlated with the amount of time the good has been on the market, as is the case in our model, then *vintage* would be a proxy for this unobserved variable (that is, rank) in the hedonic regression. Since $v_{i,j}$ is specific to the consumer group and the good, it is analogous to including $b^{i,r}$ in the hedonic regression.

If, however, the econometrician has access to consumer data, she could proxy for the premium with demographic variables such as *income*. That is, since γ is uniquely identified in the model by either the consumer type or the unobserved component of the good as in equation (11), one can control for its effects on prices by controlling for either the consumer type, i, or a specific attribute of the good that identifies rank (such as vintage). If we were to instead include the demographic variables when creating an expectation of the price paid, the expected price paid by consumer type i is given by:

$$h^r(z^i, x^j) = E[p^{i,j}|z^i, x^j], \tag{23}$$

where z^i is a vector of demographic variables that takes into account consumer i's preference ordering. This specification allows for more than one predicted price for two goods with the same bundle of characteristics. The resulting hedonic Laspeyres index for consumer i using either h^r equal to $h^r(z^i, x^j)$ or $h^r(v^{i,j}, x^j)$ is:

$$I^{h^r}(P_t^i, P_{t+1}^i) = \frac{\sum_{j\in C_t^i} h_{t+1}^r q_t^{i,j}}{\sum_{j\in C_t^i} h_t^r q_t^{i,j}} = \sum_{j\in C_t^i} \omega^{i,j} \frac{h_{t+1}^r}{h_t^r}, \tag{24}$$

where $\omega^{i,j} = \frac{h_t^r q^{i,j}}{\sum_{j \in C_t^i} h_t^r q^{i,j}}$ and represents the expenditure share of good j in consumer i's basket, C_t^i. The index for the entire population can be represented as a population weighted average of each demographic group's index, or "segment-hedonic index":

$$I^{h^r}(P_t, P_{t+1}) = \sum_{i \in N} \gamma_i I^{h^r}(P_t^i, P_{t+1}^i), \tag{25}$$

where $\gamma_i = \frac{\sum_{j \in C_{i,t}} h_t^r q_{i,j,t}}{\sum_i \sum_{j \in C_t} h_t^r q_{i,j,t}}$ represents the expenditure share of consumer group i. Unlike the original hedonic index, equation (24), $I^{h^r}(P_t, P_{t+1})$ takes into account the fact that consumers in the same demographic group, i, may make similar purchases according to some characteristic of the good that is unobservable to the econometrician.

For example, in the case of no constant in the hedonic regression the segment-hedonic prices for period t and $t+1$ are $h_t^r(z^i, x^j) = \hat{\alpha}_t z^i + \hat{\beta}_t x^j$ and $h_{t+1}^r(z^i, x^j) = \hat{\alpha}_{t+1} z^i + \hat{\beta}_{t+1} x^j$, respectively. As long as $\hat{\alpha}$, the fixed effect that controls for consumer demographics, is relatively constant across periods of time, the segment-hedonic price relative, $\frac{h_{t+1}^r}{h_t^r}$, measures the change in the predicted measure of price per quality of good purchased by consumer-type i, $\hat{\beta}$. Thus, the segment-hedonic price relative inherently controls for the premium charged to consumers of a certain demographic group.

6 Index Estimation

6.1 TUP Index

We create hedonic cost-of-living indexes with the TUP data under the index specifications described in the previous section and depict them in Table 3. The table shows estimates of the Laspeyeres index under specification (24) using equation (15) as the hedonic function. We call this the "standard hedonic" in the table. We also show estimates under specification (25), where we substitute income for $z_{i,t}$ in calculating the hedonic function, h^* as in equation (23). We call this latter specification "hedonic with income."[27]

[27]Indexes that included all of the demographic variables included look very similar to those that just include income.

The standard hedonic regression we run is that of the logarithm of price on the logarithm of speed, ram, hard drive capacity, as well as dummy variables representing brand, form, and various peripherals included on the survey form.[28] We also include dummies which represent the state in which the computer was purchased. The base hedonic with income regression includes all of the base variables plus dummy variables representing 17 income bins. We use a weighted regression using the computer weights provided to us by Metafacts, which were also used to calculate the expenditure weights.[29] Standard errors of the indexes were calculated using a residual bootstrap in which residuals of the hedonic regression were resampled with replacement.[30]

Table 3 and Figure 5 show the indexes over the five year sample period. Although the standard errors are relatively large, the results show that the index including income in the hedonic regression is larger in all but one year. This one year anomaly may be due to the recession which caused high income consumers to become more sensitive to price. The average weighted hedonic with income shows that prices fall, on average, 8.6 percent annually (that is, 100 minus 90.4) while the average base hedonic shows that prices fall 9.6 percent annually (that is, 100 minus 91.4). Thus, prices fall on the magnitude of one tenth faster (i.e. 1 divided by 9.6) when not controlling for income in the weighted regression. To test that the difference between the two indexes are significant, we took a bootstrap sample with replacement of 1000 price indexes created using the residual bootstrap method for the standard hedonic, and then repeated this for the segment

[28]The peripherals included were all dummy variables indicating the whether or not the peripheral was included. The variables are docking station, USB Hub, firewire, PCMCIA card, tape drive, CD, DVD, CDR/CDRW, and DVDR/DVDRW.

[29]MetaFacts projects survey computer sample results that are representative of the entire U.S. market for a specific household type. The first-stage weighting adjustment was made to compensate for the varying response rates of the follow-up comprehensive surveys and for over-sampling of non-using households. The second-stage weighting was made to project sample results to the entire population of U.S. households. The third-stage weighting adjustment was made to bring the data more into line with BLS estimates for total adult and total employed adult populations.

[30]The residual bootstrap works as follows: In the first stage, the hedonic regression for period t is run. The residuals are collected and sampled with replacement to create simulated prices, $\hat{p}^* = h + \mu^*$, where μ^* is a given residual sampled from the distribution of residuals. 1000 bootstrap samples of simulated prices were collected, in which 1000 price indexes could be created for each time period. The standard deviation of the distribution of indexes is then reported.

hedonic. Of those sampled using the segment hedonic, 768 (that is, 77 percent) resulted in total price declines that were less steep than the standard hedonic.

6.2 NPD Index

We now look at hedonic indexes with the NPD scanner data. This dataset has an advantage over the TUP data because there is less measurement error in both the sampling distribution of the computers as well as the reported price and attributes of the computer. The disadvantage is that we do not have demographic variables. However, we do know the age of the computer (or its *vintage*), which is a characteristic of the computer that is potentially correlated with the product's rank and subsequently consumer characteristics. Thus, we can form hedonic indexes using the hedonic function in equation (22) where we proxy $v_{i,j}$ with vintage.

Comparable to the TUP indexes, we calculate "standard hedonic" indexes as well as "standard with vintage" hedonic regressions in which we include dummy variables representing the computer's time on the market (in months). The base hedonic regression is that of the logarithm of price on the logarithm of speed, ram, hard drive, display size, and dummies indicating brand, form (that is, laptop, desktop, etc.), operating system, and other peripherals included with the computer.

Table 4 presents estimates of a matched-model, baseline hedonic, and the baseline hedonic with vintage dummies.[31] We report the average of the monthly price index for each type of price measure. The table shows that, similar to including income in the hedonic regression, including vintage in the price measure shows that prices are falling less rapidly. Specifically, the average monthly price fall is 3.8 and 3.6 percent under the matched-model and hedonic, respectively, while the monthly price fall is is approximately zero on average under the hedonic vintage Laspeyres index. Figure 6 shows the NPD price index under all three measures. While the standard-hedonic and matched-model indexes overlap and both show rapidly declining prices, the hedonic which controls for vintage shows the resulting price level is *higher* by the end of the sample period. As we did with the TUP index, we test that the standard hedonic and the hedonic with vintage

[31] All indexes use Laspeyres weights. Results did not change when using Paasche weights. See the appendix for more detailed results.

are significantly different. Of the 1000 sampled boot strap indexes using the hedonic with vintage, 990 (that is, 99 percent) resulted in total price declines that were less steep than the standard hedonic. The different results using the TUP and NPD indexes are likely attributed to measurement error in the survey as well as the fact that we are using a different proxy for consumer heterogeneity in each.

7 Conclusion

Price measures based on either a matched-model or hedonic methods show significant cost-of-living declines for personal computers. Our model shows that these cost-of-living declines will be overstated if firms are setting prices in order to separate the market between consumers with differing price elasticities or differing tastes for a product's relative position on the quality ladder. This result is especially compelling because our empirical work is consistent with such separation actually occurring in the market. Specifically, we find that higher income consumers pay a higher price than do lower income consumers for computers of the same underlying quality. Furthermore, scanner data from the NPD show that for a given computer model, nominal price falls over the life of the good.These two empirical findings point to the fact that higher income consumers purchase earlier in the life of the product when the markup is highest.

Our model shows that one should control for consumer heterogeneity when constructing price indexes and predicts that a standard matched-model and hedonic price measures will understate inflation. Our empirical findings correspond with these theoretical predictions as we find that cost-of-living indexes which control for consumer income and the age of the product show less dramatic falls in prices in comparison to measures based on the standard matched-model and hedonic measures. Controlling for a consumer's income in the TUP survey data, we find that prices fall approximately one tenth as much as compared to the standard hedonic measure without income. Furthermore, when controlling for the age of the computer in the scanner data, we find that prices are rising over the sample period as opposed to declining.

There is ample room for future analysis on this topic. While the segment indexes constructed in this study are suitable for demonstrating the potential impact of consumer heterogeneity on a cost-of-living index, such household survey information is not readily available to the BLS. Furthermore, including the age of the computer in the hedonic regression may not necessarily be a suitable fix if manufacturers are introducing new computers not at the frontier technology. Thus, developing a more intricate estimation technique to better capture consumer segments in existing BLS data seems like a worthwhile endeavor for future research.

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A Proof of Theorem 1

Proof. We start with the condition:

$$p_t(x_r) \leq \frac{1-\beta(1+\sigma^f)}{1-\beta(1+\sigma^p)} f(x_r) \tag{26}$$

$$\leq \delta f(x_r) \tag{27}$$

We must first find conditions of the parameters β, σ^f, and σ^p in which $p_t(x_r) \geq 0$. It follows that:

$$p_t(x_r) > 0 \Rightarrow \delta > 0 \Rightarrow \sigma^f, \sigma^p < \frac{1-\beta}{\beta} \text{ or } \sigma^f, \sigma^p > \frac{1-\beta}{\beta} \tag{28}$$

which means that both growth rates have to either be above or below the threshold $\frac{1-\beta}{\beta}$. We now show that it is never optimal for the firm to set the growth path of prices, σ^p, different from σ^f. We start under the condition that $\sigma^f, \sigma^p > \frac{1-\beta}{\beta}$ which means that $1-\beta(1+\sigma^p)$ is positive and increasing in σ^p. Given its current price, $p_t(x_r)$, if the firm increases the growth rate such that $\sigma^p > \sigma^f$, then $1-\beta(1+\sigma^p)$ will increase, which subsequently causes the upper bound on the price level to rise. The firm will therefore deviate from this strategy by raising $p_t(x_r)$ which lowers σ^p. If the firm instead lowers the growth rate such that $\sigma^p < \sigma^f$, then the upper bound of the price level will decrease and the firm will be forced to lower $p_t(x_r)$ which will raise σ^p. Therefore, the only stable path is $\sigma^p = \sigma^f$.

We next look at the problem when $\sigma^f, \sigma^p < \frac{1-\beta}{\beta}$, which means $1-\beta(1+\sigma^p)$ is negative and approaching zero as σ^p increases. In this case, if the firm raises σ^p given $p_t(x_r)$, such that $\sigma^p > \sigma^f$, $|1-\beta(1+\sigma^p)|$ will fall as it approaches zero. The upper bound of the price level will therefore fall, forcing the firm to lower $p_t(x_r)$ and subsequently raising σ^p even further. The firm will therefore lower prices until they hit zero. If the firm lowers σ^p such that $\sigma^p < \sigma^f$, $|1-\beta(1+\sigma^p)|$ will increase as it approaches ∞. Therefore, the upper bound price level will rise, inducing the firm to raise $p_t(x_r)$ will lowers σ^p even further. Again, the only stable path is $\sigma^p = \sigma^f$.

□

Table 1: Average Monthly Price Changes

	Avg. Monthly Price Change
Toshiba	-0.039
Sony	-0.034
Emachines	-0.033
Hewlett Packard	-0.030
Compaq	-0.028
Apple	-0.002

Note: Price change averages are based on total units sold for each model over the first six months on the market: $\bar{\Delta} P_b = \frac{\sum_{j \in b} \frac{\sum_1^{T_j} \Delta P_{i,t}}{T_j} q^j}{q_b}$, where $\Delta P_{j,t}$ is the monthly change in price of model j (that is,, $\ln \frac{P_{j,t}}{P_{j,t-1}}$), q^j is the total quantity of model j, and q_b is the total quantity of brand b (that is,, $\sum_{j \in b} q^j$).
Source: NPD

Table 2: Technology User Profile: Fixed Effects Regressions

	(1)	(2)
lnIncome	0.073***	0.087***
	(0.024)	(0.028)
Age55-59		-0.063
		(0.060)
Age50-54		-0.088*
		(0.053)
Age45-49		-0.084*
		(0.049)
Age40-44		-0.037
		(0.050)
Age35-39		-0.088
		(0.054)
Age30-34		-0.081
		(0.059)
Age25-29		-0.062
		(0.067)
Age20-24		0.089
		(0.089)
SomeCollege		-0.031
		(0.043)
College		-0.007
		(0.047)
Graduate		-0.095*
		(0.050)
EducationNA		0.027
		(0.072)
Black		0.090
		(0.087)
Asian		-0.008
		(0.125)
Other		0.057
		(0.064)
Single		-0.045
		(0.046)
Observations	1502	1502

Note: The dependent variable is the natural logarithm of price for both fixed-effect regressions (1) and (2), where the fixed effect is an indicator of the year-RAM-speed-harddrive-form-manufacturer. As the TUP survey does not indicate the size of the monitor purchased, we limit our sample to desktop computers purchased with no monitor.

Table 3: Technology User Profile Hedonic Indexes

	Standard Hedonic	Hedonic with Income
1999 to 2000	0.898	0.921
	(0.058)	(0.067)
2000 to 2001	0.960	0.965
	(0.039)	(0.042)
2001 to 2002	0.890	0.881
	(0.038)	(0.039)
2002 to 2003	0.869	0.887
	(0.040)	(0.046)
Average	0.904	0.914
	(0.044)	0.048

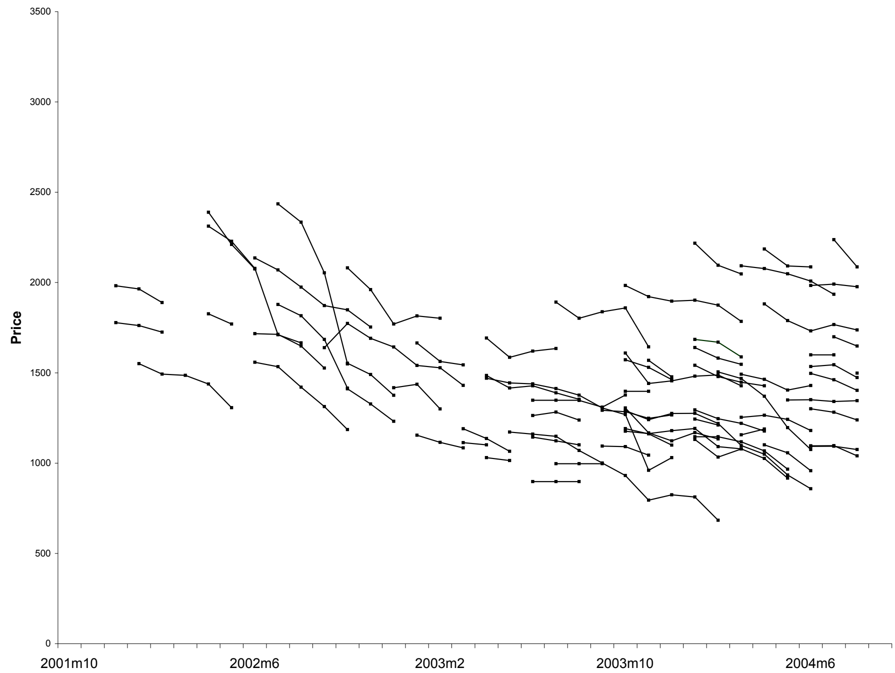
Note: The hedonic index is based on a regression of the logarithm of price on the logarithm of speed, ram, hard drive capacity, as well as dummy variables representing brand, form, and various peripherals included on the survey form. The hedonic with income includes income in the regression. Standard errors were calculated using a residual bootstrap in which residuals of the hedonic regression were resampled with replacement.

Table 4: NPD Hedonic Indexes

	Index (Averages)
Matched Model Laspeyres	0.962
	n/a
Hedonic Laspeyres	0.964
	(0.036)
Hedonic Vintage-Laspeyres	1.007
	(0.043)

Note: Averages of the monthly price changes for the sample period are reported. The hedonic index is formed from a hedonic regression of the logarithm of price on the logarithm of speed, ram, hard drive, display size, and dummies indicating brand, form (that is, laptop, desktop, etc.), operating system, and other peripherals included with the computer. The hedonic-vintage includes age-of-computer (that is, vintage) dummies. Standard errors were calculated using a residual bootstrap in which residuals of the hedonic regression were resampled with replacement.

Figure 1: Price of Hewlett Packard 15-Inch Laptop Computers



Note: Depicted are the price contours of all 15 inch notebook computers sold by Hewlett Packard. For ease of view, prices after the the units CDF reached 90 percent for each model were omitted. Source: NPD

Figure 2: Generating Price Contours

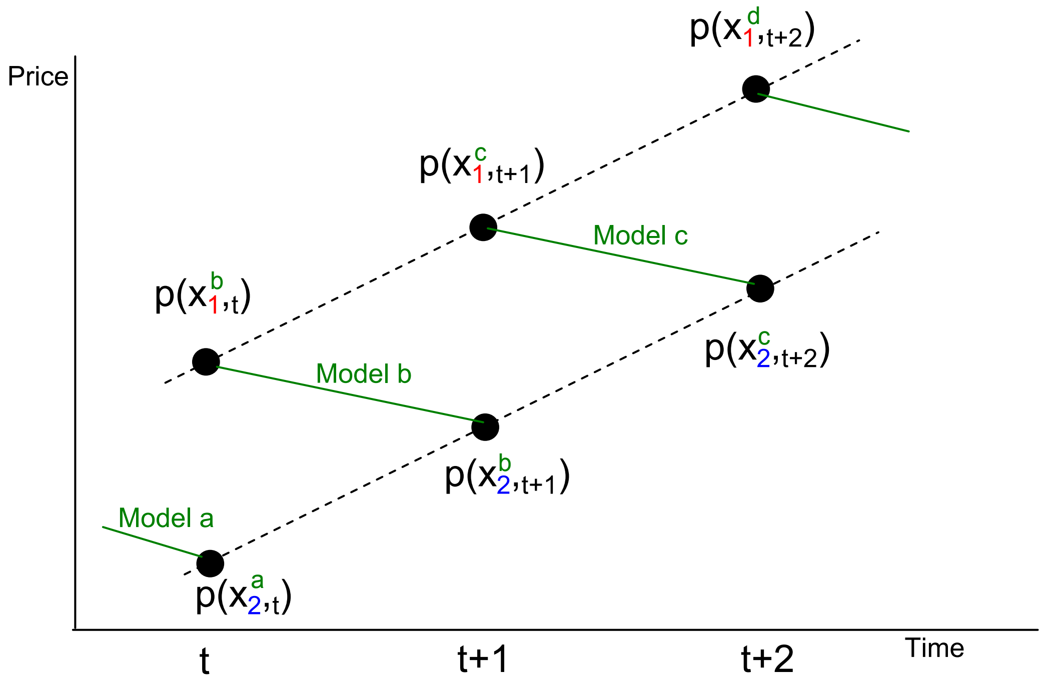


Figure 3: Decomposition of Premium

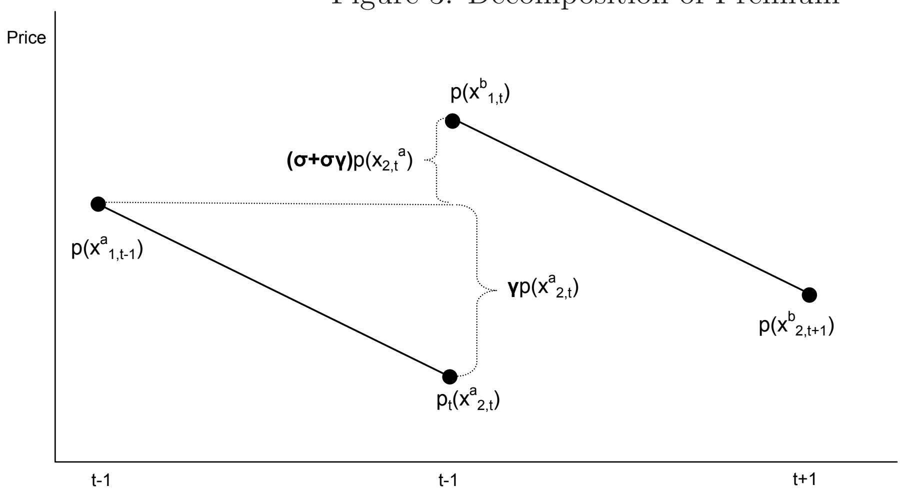


Figure 4: Converting the Axis to Price versus Quality

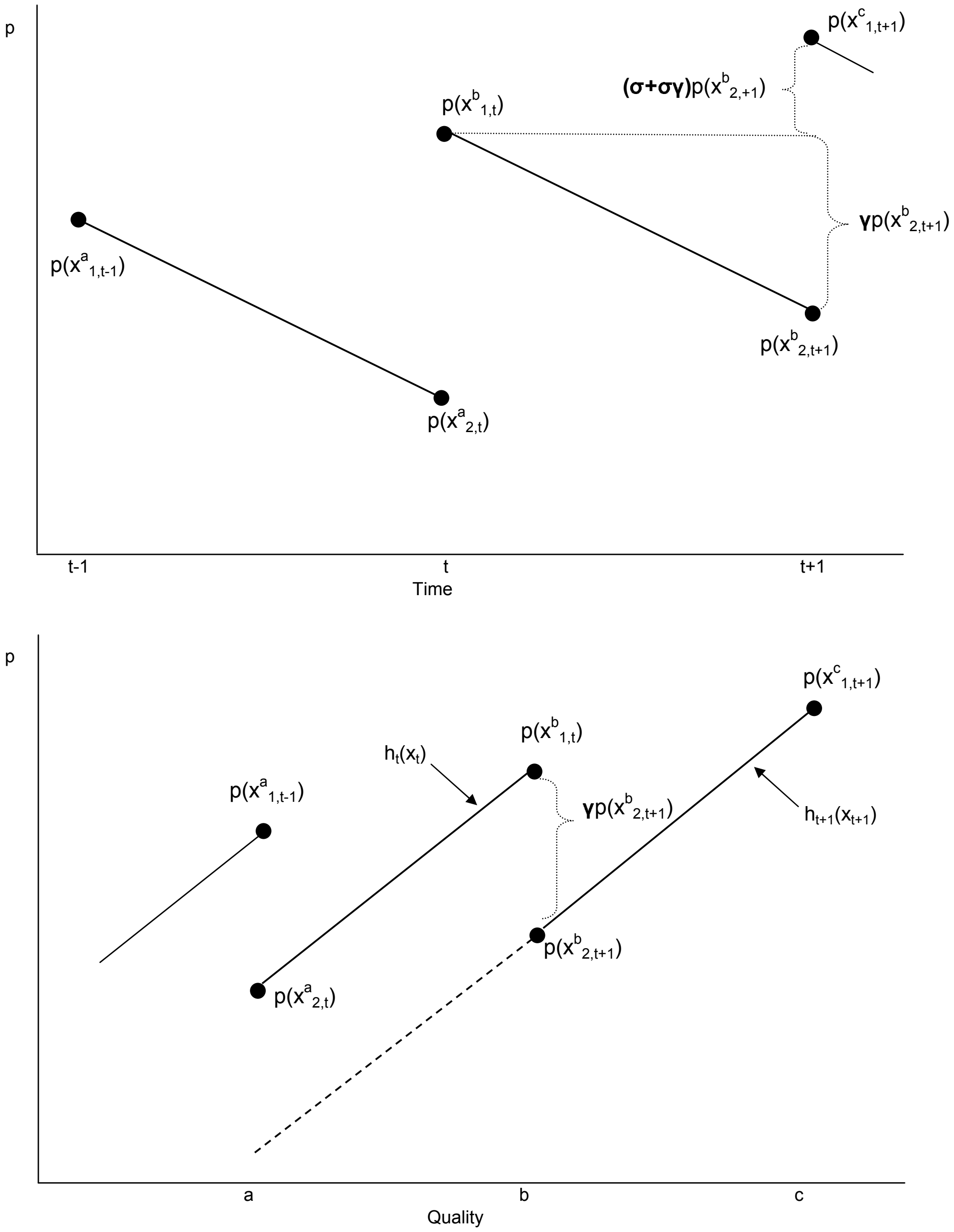


Figure 5: Technology User Profile Hedonic Indexes

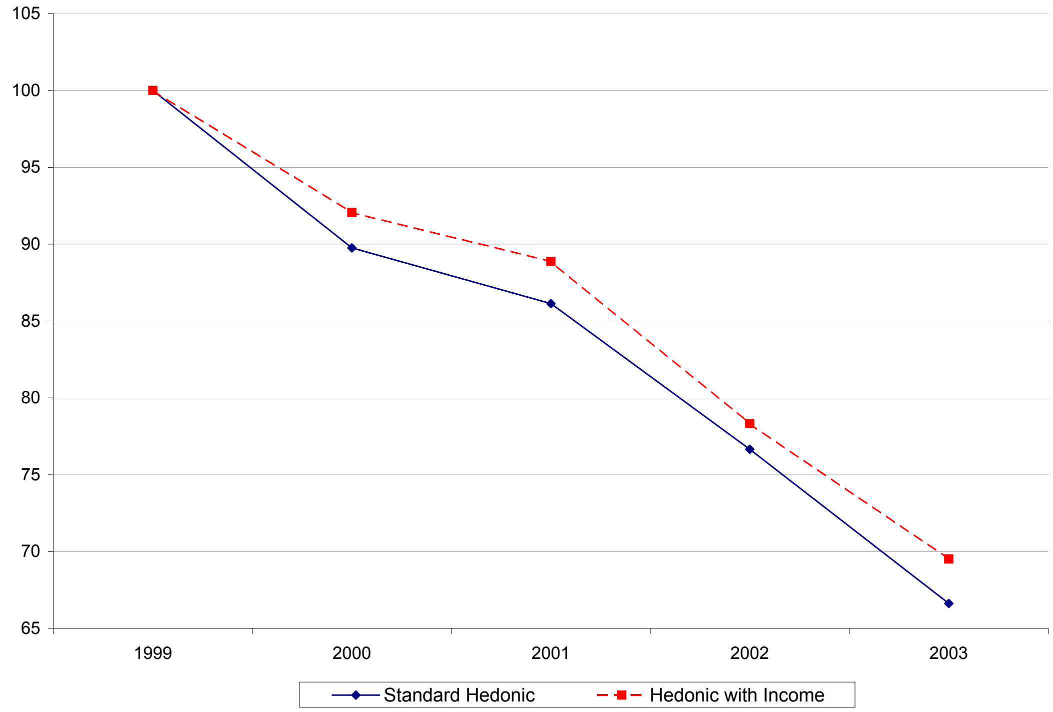


Figure 6: NPD Hedonic Indexes